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Essays On Search

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Abstract

This dissertation consists of three chapters that examine search frictions within the macroeconomy. In the first chapter, I construct a model of simultaneous search to propose a novel contributor to the twin effects of labor force participation decline and rising wage inequality. An algorithm for solving the pairwise-stable matching in a macro environment is provided and incorporated into a dynamic, general equilibrium model. Falling search costs will generate falling labor force participation—as the lowest ranked workers are crowded out of the market—and rising wage inequality—as the competition for desired skills increases. An empirical tests corroborates the effects of cheap search on falling participation.

Chapter 2 assesses the contribution of aggregate vs. sectoral shocks to output volatility by building a real business cycle model calibrated to match realistic structure in the market for all three major inputs to production---material inputs, capital goods and labor. While the former two inputs are standard, this paper innovates on previous methods by first expounding the structure of sectoral labor reallocation and then calibrating a model to match its features. A common-factor estimation procedure attributes approximately half of aggregate output volatility to sectoral shocks.

In Chapter 3, I explore the implications of lower search costs for product markets by building a micro-founded model of shopping within an industry that features realistic product search frictions. I show via a precise characterization that either increasing or decreasing prices in response to cheaper search can be consistent with competitive equilibrium, depending on the distribution of consumer tastes. This distribution-dependence further dictates whether firms and varieties enter or exit the marketplace.

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Dedicated to all of those who believed in me, especially my parents.

ABSTRACT

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Francis DiTraglia

This dissertation consists of three chapters that examine search frictions within the macroeconomy. In the first chapter, I construct a model of simultaneous search to propose a novel contributor to the twin effects of labor force participation decline and rising wage inequality. An algorithm for solving the pairwise-stable matching in a macro environment is provided and incorporated into a dynamic, general equilibrium model. Falling search costs will generate falling labor force participation—as the lowest ranked workers are crowded out of the market—and rising wage inequality—as the competition for desired skills increases. An empirical tests corroborates the effects of cheap search on falling participation.

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Contents

List of Tables

List of Figures

Chapter 1

Wage Inequality and Labor Force Participation in a Simultaneous Random Search Environment

1.1 Introduction

An increasing proportion of job search and hiring now occurs online. According to the survey SilkRoad (2017), in 2017 over 50% of interviews are now sourced through online contacts. In this same survey, the conversion rates of external application to interview and external interview to hire are 33:1 and 3:1 respectively, combining to suggest that every vacancy that is filled through online sources will require on average $\sim$ 99 applications. Similarly, Marinescu and Wolthoff (2015) find using data from CareerBuilder.com that there were 59 applications per vacancy on that website. Compare this number to that reported in Faberman and Menzio (2018) from the 1982 Employment Opportunity Pilot Project who found an application per week per vacancy statistic of 22.3 for those vacancies filled within one week (and notably fewer applications for all other openings.)

Corresponding with the rise in online job search has been a marked decrease in labor force participation. As seen in 1.1, from its peak above 67% in 2000, participation has fallen to under 63% in 2018.

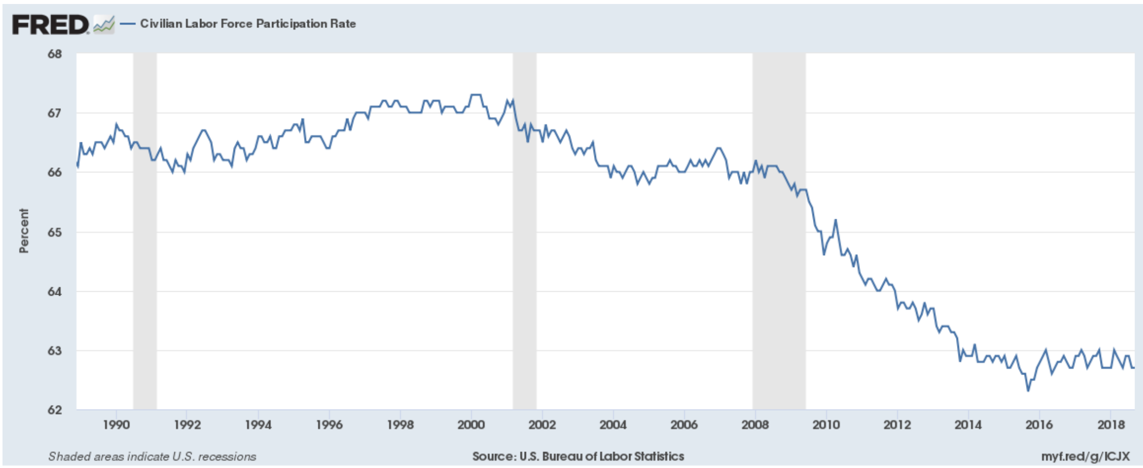


Figure 1.1: Labor Force Participation

Yet this transition in labor force participation has not occurred equally for all groups. According to Frazis (2017), the two single biggest predictors of labor force exit over the period 2005-2015 were education and age of the worker–with younger, less-educated workers exiting the labor force in much higher quantities.

As I will suggest later in the paper, labor force exit by predominantly younger and lower skilled workers can be explained by a permanent "crowding out" effect brought on by a rise in search efficiency. In a low information environment–where firms come into contact with few workers–the lowest skilled in any distribution have a chance of "getting lucky", and being the best candidate around for the job. In the alternative–when firms sample a large amount of potential workers–it is extremely unlikely that this same worker will prove to be the best candidate for any job. Hence, their motivation to search will dry up and they will exit the labor market. In a sense, workers of low skill benefit from search frictions, in that they can be in the right place at the right time with a poorly informed firm that might otherwise have matched with a better worker.

This notion of "crowding out" is an old concept in the job search literature, though usually cast in the context of movements over the business cycle. As labor conditions worsen, firms can increase their hiring standards due to an increased pool of workers. To my knowledge, this is the first paper that suggests this effect being generated by long-run changes in search technology as opposed to productivity movements over the business cycle.

Concomitant with a fall in labor force participation has been an increase in wage inequality. Pew Research (2018) documents that real wage increases since 2000 for the lowest and highest decile of earners have been 3% and 15.7% respectively. The current paper suggests a novel contributor to this rising wage inequality: as worker-firm contacts increase–that is, as search frictions decline–competition for workers will increase and the incentives to post high wages will increase disproportionately for already high-paying firms.

In the wage-posting environment I will build, posting a higher wage gives a notion of preference to a firm in their worker choice process. When firms are more informed about the existence of good workers they can choose more discriminately, and the value of this choice preference increases along with the incentive to raise wages. The nature of this incentive is predominately a competitive one–deviations yield preference at the cost of others–and so higher-ranked firms will be disproportionately affected, as they face more competition "from below" than lower-ranked firms. Higher-ranked firms will be induced to raise their wages, and higher-ranked workers will more efficiently match with them.

The model used to demonstrate the twin effects of crowding out and wage inequality will feature simultaneous random search. Workers will be allowed to sample firms randomly at will; but, they will face a cost to do so. Time will be discrete, and the competition amongst firms and workers in the matching space will be made explicit–firms will be contacted by multiple workers and will choose the best candidate willing to work for them, given that worker's other contacts–i.e. a pairwise-stable matching. The pairwise stable matching will occur in the context of (and be enabled by) wage posting with full commitment.

I will demonstrate, under mild assumptions, that the solution of the pairwise-stable matching problem is tractable and it is governed by the solution of a system of two first-order partial differential equations. In equilibrium, the solution of this system pins down the expectations of workers and firms about their prospects of participating in the matching market. I will approximate the solution to this system via pseudospectral element methods with an endogenous division of the subdomain. In a technical sense, this paper will be of interest to those who wish to solve such problems–a common occurrence when distributions must be solved numerically within a model with actionable regions. The pseudospectral methods must be implemented on subdivisions of the matching region because the model will feature nondifferentiabilities and occasionally discontinuities–a fact that I will document.

After the model exposition, I will provide evidence of a link between labor force participation and cheap search in the microdata of the Consumer Population Survey from the BEA. Within an econometric specification, cheap search will be identified through the Computer and Internet Use Supplement to the CPS survey, which asks users about internet job search. A logistic regression specification will demonstrate a strong statistical link between the presence of internet search and labor force exit by specifically younger and uneducated workers—which represent two proxies for worker rank.

This paper joins a large and growing literature on matching environments with random search frictions. Most of this literature focuses on sequential search (see: Shimer and Smith (2000) for a pioneering example and Chade et al. (2017) for a thorough review.) I have chosen to model simultaneous search here for a few reasons.

The first is that it more accurately captures the decision problem faced by firms. Generally firms receive a slew of applications, they sort through them and make offers in order of their preferred candidates. This logic is more closely mimicked in a simultaneous search environment and so allocations are likely to be better represented that way. Obversely, in a sequential search model, a firm is merely looking for the first acceptable candidate to arrive, and this acceptability is determined by a somewhat nebulous concept of "discounting". It is unclear, therefore, that the hiring decision of firms in a sequential model would match up–especially as the rate of contacts increases to many per day–to the intuitive picture of a firm sorting through applications.

The second is that search competition in a discrete time setting is more amendable to calibration and estimation. Data are received in discrete form; and indeed, continuous time models are discretized vis-a-vis time in order to be fit to the data. Simultaneous search therefore provides a ready form to be fit to data, as well as to serve as a check on the prescriptions of the numerous models discretized from continuous time.

Another major contrast between this paper and the broader literature is the use of wage-posting. A majority of models of search and matching rely on a bargaining game to solve the bilateral monopoly problem of a matched firm and worker. Bargaining serves to lend a great deal of tractability to what are otherwise complicated models. The decision to use wage posting in this paper was made for two reasons. First, wage-posting generally characterizes the behavior of firms looking to hire for low-skilled positions in the US labor market. Therefore, wage-posting is a more natural choice when modeling the behavior of lower-ranked workers. Additionally, wage-posting serves as a way to clear the matching market in a simultaneous search and matching setting.

Several related papers to the current one are as follows. Uren and Virag (2011) build a wage posting model in a search environment in continuous time. Their goal is to demonstrate how changes in the production function–notably via skill-biased technological change–can induce rising wage inequality. The current paper focuses instead on how the changing search technology can generate effects on labor force participation and wage inequality.

There is also an emerging literature on the increasing efficiency of search or, equivalently, in falling search frictions. Martellini and Menzio (2018) find evidence that search frictions have declined considerably in recent years, and provide a theoretical rationalization of how stationarity of key labor statistics can be preserved in spite of falling search costs. In another vein, Hursey (2018) shows that in a product search environment, increasing search efficiency can have positive or negative effects on both prices and market entry depending on the shape of idiosyncratic tastes. The current paper will employ a similar mechanism to the case in Hursey (2018) where idiosyncratic tastes are small in magnitude: the competitive effects of search will dominate and force the lowest performers out of a market.

The remaining paper will be organized into seven sections. Section 2 will present the model environment, followed by the pairwise stable matching algorithm in Section 3. Section 4 will embed the matching space into a general equilibrium framework, while Section 5 will provide details on the computation of the model via pseudospectral methods. Section 6 will show numerical results and Section 7 will include empirical ones. Section 8 concludes.

1.2 Model Environment

The model is set in discrete time with infinite periods denoted by $t \in \{0, 1, ...\}$. There are two types of agents: workers, who are endowed with an indivisible unit of labor and an idiosyncratic skill level a; and firms, to whom the workers' labor can be sold and who own their own technology characterized by the function $y : A \times X \to \mathbb{R}$ which maps their efficiency level x and the ability of their worker a into an output quantity y.

The workers constitute a positive measure of agents, normalized to 1. Of those, e are matched with the same number of firms at any given time while $1-e$ are unmatched. There is a positive measure V of unmatched firms/vacancies which is composed of a measure of different efficiency firms such that $\int_X v(t)dt = V$. Therefore, the distribution of efficiencies of actively searching firms can be given as $F(x) = \frac{\int_{\underline{x}}^{x} v(t)dt}{V}$.

The distribution of the abilities of workers in the population as a whole is described by the CDF $\Omega : A \to [0, 1]$ which gives the probability $\Omega(a)$ that a randomly drawn worker from the overall population will exhibit less ability than a. The efficiencies of firms are also described by a CDF $\Lambda : X \to [0, 1]$. This CDF describes the distribution of efficiencies of *putative* firms; that is, those firms who could potentially enter the matching market should they choose. The measure of new vacancies introduced each period which draw from these efficiencies is given by $\bar{v}$.

Each firm and worker can be of two types: matched; or unmatched. At the beginning of the period production occurs and matched worker-firm pairs produce using their technology while unmatched workers engage in home production that yields a payoff $b \in \mathbb{R}_+$. Matched workers are paid a contracted wage w that is unchanged over the life of the match. Unmatched firms produce nothing.

After production, unemployed workers can engage in costly search in order to match with an unmatched firm and begin employment. There is no search on the job but it would be an interesting direction for future research. Workers own a search technology $z : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ that allows them to convert costly effort into an *expected* number of connections with unmatched firms. The mechanism by which connections are converted into consummated matches will be described later in the section on the matching algorithm.

The unmatched firms over which the unemployed will search are operating a vacancy with which they have been endowed. At the end of the period, unmatched firms' vacancies have a probability δ_f of exploding, while a consummated match has a probability δ of doing so. As separation occurs after search, separated workers do not engage in it in the period they lose their job.

1.2.1 The Search Process

As stated in the previous section, workers can exert effort to match with z number of expected vacancies. This subsection will describe the manner in which those connections are apportioned amongst firms. This process will take the form of growing a bipartite graph between the measures of firms and workers within the model.

The very first action in the search phase of the model is wage posting. Firms post a wage $w \in \mathbb{R}$ to which they are committed during the period–meaning it is unchangeable throughout the process of search and matching and must be paid over the life of any consummated match. The distribution of these wages is given by the CDF $F_w(w)$ which is both endogenous and of common knowledge. Upon meeting a firm, an unemployed worker observes its posted wage.

Let $z_t(a)$ represent the expected number of connections obtained by the worker of ability a. The actual number of connections of this worker is a random variable described by the discrete distribution

$$\Theta_0(a) : \mathbb{Z} \times \mathbb{R}_+ \to [0, 1]$$

. For the remainder of this paper I will assume than this distribution is a Poisson distribution parameterized by rate $z_t(a)$. A Poisson distribution will deliver more tractable quantities for the matching process; though other distributions may be used.

Let the measure of unemployed workers of ability a at the beginning of a period be given by $u_t(a)$. Then the total measure of connections generated by the unemployed at the beginning of the period is given by the quantity:

$$Z \equiv \int_{s \in A} z(s) u_0(s) ds$$

. These connections need to be distributed over the total measure of vacancies V.

The allocation of connections is at its core an example of generating a bipartite graph between a positive measure U_t of unemployed and V_t of vacancies. Let $\Sigma = (U, V, \varepsilon)$ describe this graph with U, V the vertices and ε representing the set of edges between them. For this graph to be consistent, a rule for distributing connections over $U \times V$ needs to be proposed such that the total number of connections equals Z_t and the number of connections possessed by both sides of the graph clear:

$$Z_t = V_t \int_{s \in W} r(s) dF(s)$$

where $r(w)$ describes average number of connections received by firms posting wage w.

For the remainder of the paper I will assume that the rule describing the edge generation of the graph is the following one. It features independent allocation over vacancies and workers, i.e. the probability that a connection belonging to a worker of ability a also belongs to a firm of wage w does not depend on a or w. Further, firms receive a random allocation of connections within wage levels that is itself described by a Poisson distribution. That is, all firms receive a random allocation of connections given by a draw from Poisson distribution with rate $r(w) = r \quad \forall w$. To sum up, the edge clearing of the bipartite graph is described by the equation:

$$Z_t \equiv \int_{s\in A} z(s)u_0(s)ds = V_t \int_{s\in W} r(s)dF(s) = rV_t$$

The method of graph generation given above which gives no preferential attachment based on agent type is in some sense the most random–or undirected–possible. The wages and abilities of workers give no guidance to which connection are formed between which agents. While this assumption might not be entirely realistic, I consider it the ideal starting point for an analysis of random, simultaneous search. Modifications which give preferential attachment based on wage or ability–rendering search somewhat directed–is both beyond the scope of this paper and a very interesting direction for future research.

1.3 The Matching Algorithm

The purpose of this section is to describe the algorithm used to generate a pairwise stable matching between unemployed workers and vacancies. First, a concept of firm ordering based on posted wage will be introduced. The algorithm is then cast in terms of movement along the queue generated by such an ordering. The initial system over the countably infinite equations of the graph degree distribution will be reduced into a system of two partial differential equations which pin down firm and worker matching prospects in equilibrium.

1.3.1 Firm Ordering

In order to execute the algorithm, a concept of firm ordering needs to be established. The ordering describes the preference given to firms in their decision-making process, meaning a higher-order firm's decision will always take precedent over a lower. Simply, the ordering of firms is determined by their wage.

Let $F(w)$ represent the CDF of wages posted by firms in equilibrium and let $w(F)$ represent its inverse. If $F(w)$ is continuous at a point w, a measure 0 set of firms is choosing and therefore a firm with wage w faces a zero probability of a rival claim to a given worker. However, if a mass point exists in $F(w)$ and two or more firms covet the same worker, it is assumed the worker chooses randomly between them. However, this cannot occur according to the following,

Proposition 1.3.1. *$F(w)$ contains no mass points*

Proof. Suppose F contains a mass point at w and consider a firm with a preferred candidate. Because there is a positive measure of firms choosing along with this firm, they face a non-zero probability p of losing this preferred candidate through randomization; while, a wage $w + \varepsilon$ would have guaranteed recruitment for any $\varepsilon > 0$. Taking expectations implies there is a discontinuous gain and $\exists \varepsilon$ that is a profitable deviation. $\square$

The matching algorithm, in order to generate a pairwise-stable matching, can be understood to progress sequentially along the ordering generated by wages. Starting at $F = 1$ and progressing downward in the rankings to 0, all firms posting a wage of $w(F)$ make their selection of their preferred candidate from amongst their available connections (if any).

The firm ranking F as a state variable in the algorithm can therefore be thought of, in one interpretation, as similar to time in a differential equation–where each period is enacted internally in continuous time progression from $F = 1$ to 0. Of course this is just an interpretation of the device used to generate the calculation of the pairwise stable matching and need not be taken explicitly.

1.3.2 Preliminaries to the Algorithm

Before describing the algorithm, several objects need to be posited which will be identified as equilibrium objects by the firm and worker problems later in the paper.

The first of these is $\underline{a}(F)$, which gives the lowest overall quality worker that a firm at queue rank F would consider hiring; that is, it is its reservation quality. Obversely, $\underline{F}(a)$ gives the lowest-ranked firm a worker of ability level a would be willing to accept. This immediately implies that $w(\underline{F}(a))$ is worker a's reservation wage.

If the production function $y(a, x)$ is increasing in a, then $\underline{F}(a)$ is increasing and therefore can be inverted for $\bar{a}(F)$, the highest ability worker willing to accept an offer from firm rank F (and wage $w(F)$). It will be convention for the algorithm that those workers whose reservation wage has passed will exit the matching market and their links will be destroyed.

Lastly, let $u(a, F)$ represent the remaining measure of unemployed workers of ability level a after all firms with efficiency greater than F have moved. And let the total number of links possessed by workers (and firms through market clearing) at a point F in the algorithm be given by the function

$$\tilde{z}(F) = \int_{\underline{a}}^{\bar{a}(F)} z(s)u(s, F)ds$$

1.3.3 The Algorithm

Given the policy rules $\underline{F}$ and $\underline{a}$ this subsection will describe the allocation that occurs from the pairwise stable matching algorithm. It will first describe the firm's side of the algorithm. Firms will possess at any given queue position a certain number of potential connections with workers. As firms above a given firm move, these connections will disappear as either other firms steal workers, or the workers exit because their reservation wage has passed.

After describing the degree distribution of active connections, I will detail the firm's problem. This will provide conditions under which the wage posted by a firm is increasing in its efficiency–meaning more efficient firms post higher wages. Therefore, efficiency, wage, and queue order are all monotonically related, and efficiency may serve as a sufficient statistic for queue order. The firm's side of the algorithm will deliver the equilibrium distribution over the *best* candidate that a firm can expect to hire from their basket.

The workers' side of the algorithm will be described after the firms'. Workers similarly will have a degree distribution, or distribution over active connections to firms. They will be hired by a given firm if their overall quality is the highest in that firm's basket, and that quality is above the firm's reservation quality. The result of the worker's side of the matching algorithm will be a partial differential equation that describes the instantaneous probability of a worker of ability a of being hired at the queue position of firm efficiency x. This will combine with the condition on firm's expectations to fully describe the dynamics of the matching algorithm.

1.3.3.1 The Firm Side

Let $\phi(t, F) = \{\phi_i(t, F)\}$ describe the probability mass of number of active connections of a firm of type t after all $\hat{F} \in [F, 1]$ firms have made their selections, also known as the conditional degree distribution. So for example, the initial distribution is given by $\phi_i(t, 1) = e^{-r}\frac{r^i}{i!}$, or the Poisson pmf.

The following proposition shows that $\phi()$ will be identical for all firms remaining in the algorithm by showing the three conditions that pin down its dynamics are identical for those firms.

Proposition 1.3.2. $\phi(t, F) = \phi(F) \quad \forall t \leq F$

Proof. Let t_1, $t_2 < F$ be two ranks lower than the currently choosing one. $\phi(t_1, 1) = \phi(t_2, 1)$ by assumption. Further, the distribution over ability of t_1 and t_2's basket is independent of their type by assumption. Lastly, firm selection of candidates does not condition on candidate's connections to other firms $\Rightarrow$

$$\frac{\partial \phi}{\partial F}(t_1, F) = \frac{\partial \phi}{\partial F}(t_2, F)$$

$\square$

As a result, separate distributions need not be considered for each firm rank. Therefore let $q(F)$ represent the common, instantaneous rate of link destruction for firms who have not moved yet, meaning the unconditional rate at which a firm's connections disappear as higher-ranked firms move. The sources of link destruction are twofold: the rate at which workers are stolen by firms of a higher wage, and the number of workers who exit the market because their reservation rank $\underline{F}$ has passed.

The evolution of the distribution ϕ can be described via the differential equation

$$\dot{\phi}_j(F) = [j\phi_j(F) - (j+1)\phi_{j+1}(F)]q(F)$$

For an infinitesimal change in the queue, each bin in the degree distribution sees an inflow from the bin above it equal to $(j+1)\phi_{j+1}(w)q(w)$ which is the rate at which the remaining firms lose their $j+1$ links times the measure in that bin. Similarly, the bin at j loses its own denizens at an analogous rate times its own measure. Putting these two together yields the differential equation above.

These differential equations over all j create a countably infinite system of linear differential equations in the instantaneous link destruction rate $q(w)$. It can be represented in matrix notation as

$$\dot{\phi}(F)' = \phi(F)' \begin{pmatrix} -q(F) & 0 & 0 & 0 \\ 2q(F) & -2q(F) & 0 & 0 \\ 0 & 3q(F) & -3q(F) & 0 \\ 0 & 0 & ... & ... \end{pmatrix}$$

which is a state-dependent, continuous-time, infinite, non-homogeneous Markov chain where the states are the bins in the degree distribution given by the vector $\phi(F)$.

Proposition 1.3.3. *Let $\tilde{q}(F) = \int_F^1 q(t)dt$. When the initial degree distribution is Poisson with rate r, the firms' degree distribution for the matching algorithm is given by*

$$\phi_j(F) = \frac{(re^{-\tilde{q}(F)})^j}{j!} e^{-re^{-\tilde{q}(F)}}$$

Proof. By direct inspection $\square$

If a more constructive proof is desired, one can use the matrix exponential form from the Markov Process. Note that the degree distribution of links over firms remains Poisson throughout the algorithm, with the rate parameter now given by $re^{-\tilde{q}(F)}$, that is, initial rate degraded by outflows.

For now, assume that there exists a distribution over ability level for all of the extant links in the algorithm at any given position F in the algorithm. That is, there exists a function $\tilde{H}(a, F)$ with support $[\underline{a}, \bar{a}]$ which gives the probability that a randomly selected connection between a worker and a firm will have a quality less than or equal to a after all firms above F have made their decision.

Consider a firm that has i active links and is making its decision about whom to hire from its available pool of i workers. The distribution describing the talent level of the best of these i workers, otherwise known as the ith order statistic, is given by $\tilde{H}(a, F)^i$.

Therefore, putting this together with the derived degree distribution for firms, the *expected* pseudo distribution from which a firm will hire its worker at queue position F conditional on having at least one connection and choosing the best available worker is given by

$$\begin{aligned} H(a,F) &= \sum_i \phi_i(F)\tilde{H}(a,F)^i \\ &= e^{-re^{-\tilde{q}(F)}} \sum_i \frac{(re^{-\tilde{q}(F)}\tilde{H}(a,F))^i}{i!} \\ &= e^{-r(1-\tilde{H}(a,F))e^{-\tilde{q}(F)}} \end{aligned}$$

The program of a firm with efficiency level x can now be given. A matched firm with efficiency x, quality q and wage w produces the following lifetime value function:

$$J_m(x,q,w) = \frac{y(x,q)-w}{1-\beta(1-\delta)}$$

Therefore, the program of an unmatched firm with efficiency x is given by:

$$J(x) = \max_{q,F} \beta \left[\int_q^\infty \frac{y(x,s)-w(F)}{1-\beta(1-\delta)} dH(s,F) + H(q,x)(1-\delta_f)J_{t+1}(x) \right]$$

The firm has two choices to make. The first involves the lowest quality match it would accept. The second is the position in the queue that the firm would like to occupy. The cost for slotting at the queue position F is that the firm must post the wage $w(F)$, or the quantile of F, that guarantees it is the Fth ranked firm in the wage distribution.

In order to make x a sufficient statistic, it is required that the decision rule $F(x)$ is increasing; that is, the queue position chosen by a firm is an increasing function of their efficiency level. The following proof establishes this as the case for a production function of the form $y(x,a) = x^\alpha a^{1-\alpha}$. Any CES production function with a lower elasticity will then obtain as well.

Proposition 1.3.4. *if* $y(x,a) = x^\alpha a^{1-\alpha}$, $F(x)$ *is increasing.*

Proof. Suppose there are $x < x'$ s.t. $F(x) = F > F(x') = F'$.

$$\Rightarrow J(x', F) > J(x', F) \text{ and } J(x, F) > J(x, F')$$

$$\iff J(x', F) - J(x, F) < J(x', F') - J(x, F')$$

$$\iff \int_{\underline{q}}^{\infty} (x' - x) s dH(s, F) < \int_{\underline{q}'}^{\infty} (x' - x) s dH(s, F)$$

$$\iff \int_{\underline{q}}^{\infty} s dH(s, F) < \int_{\underline{q}'}^{\infty} s dH(s, F)$$

$$\iff J(x, F') > J(x, F) \quad \Rightarrow\!\Leftarrow$$

□

.

Given a function that ensures a monotonic queue decision rule, the firm's efficiency can then serve as the sufficient statistic for the state of both the wage distribution and the queue order as the algorithm progresses. Thus, take $w(x) = w(F(x))$ to be the wage posted by firm x, $F(x)$ to be the queue position of firm x, and $H(a, x) = H(a, F(x))$ to be the quality pseudo-distribution that firm x guarantees itself by obtaining queue position F.

The firm's side of the algorithm delivers the distribution $H(a, x)$ that is faced by the average firm of a given efficiency x. The next section will use this equilibrium object to determine how workers outflow from the market. It will first demonstrate that the object $H(a, x)$ is an exponential of the total mass of links with quality above the worker's ability a. Then, it will use this fact to deliver a two-equation system of partial differential equations that determine the instantaneous probability of being hired throughout the algorithm.

1.3.3.2 Workers

Workers who are still in the market maintain a set of active links from which they could potentially be hired. On the other end of any of these links is a firm also connected to other workers.

Consider a given position in the queue, x. At that position, an instantaneous proportion of $\frac{f(x)}{F(x)}$ links are being actively considered by firms. Now consider a worker of ability a amongst those being actively considered. Then the probability that the worker is the preferred candidate is

$$\frac{\sum_{i\geq 1}\phi_i(x) i\tilde{H}(a,x)^{i-1}}{\sum_{i\geq 1}\phi_i(x)} = e^{-r(1-\tilde{H}(a\xi,x)e^{-\tilde{q}(x)}} = H(a,x)$$

The worker considers the distribution over how many other links the firms might be connected to, and the probability that any of those other connections is owned by a worker of higher ability. Perhaps unsurprisingly, this hiring probability is exactly the first order pseudo-distribution that firms expect when entering the market.

Let $p(a,x)$ represent the instantaneous probability of any of a worker's links being converted into a job offer, this probability is given as

$$p(a,x) = \begin{cases} \frac{f(x)H(a,x)}{x} & a \geq \underline{a}(x) \\ 0 \quad \text{o.w.} \end{cases}$$

with the complementary rate that a link does not yield a hire and is destroyed given by:

$$\gamma(a,F) = \begin{cases} 1 - \frac{f(x)H(a,x)}{x} & a < \underline{a}(x) \\ 1 \quad \text{o.w.} \end{cases}$$

Then the following differential equations represent the measure of unemployed workers of ability level a with j active connections to firms remaining in the algorithm:

$$\dot{m}_j(a,x) = jm_j(a,x)(p(a,x)+\gamma(a,x)) - (j+1)m_{j+1}(a,x)\gamma(a,x)$$
$$m_j(a,\bar{x}) = u_t(a)e^{-z(a)}\frac{z(a)^j}{j!}$$

workers see outflow from having j connections equal to the sum of the rates of link destruction and hiring, and at rate $\gamma(a,x)$ they have a link destroyed and flow into the j connection bin. The solution of this differential equation is given by

Proposition 1.3.5. *$m_i(a,x)$ is given as*

$$m_i(a,x) = u_t(a)\frac{(z(a)F(x))^i}{i!}e^{-z(a)\left[F(x)+\int_x^{\bar{x}(a)} f(t)H(a,t)dt\right]}$$

Proof. By direct inspection $\square$

summing up the measure of workers over all bins yields the desired measure of workers remaining of ability a at a point x in the algorithm:

Proposition 1.3.6. *$u(a,x)$ is given by*

$$u(a,x) = u_t(a)e^{-\frac{z(a)}{v}\int_x^{\bar{x}(a)} v(t)H(a,t)dt}$$

The remaining measure of unemployed of ability a outflow at a rate given by the expected number of links they received, times the product of their rank in the first order distribution $H(a,x)$ with the probability mass of vacancies at x.

As stated previously, at any point in the algorithm, the number of links possessed by firms and workers must add up, this can now be precisely given by the following condition:

$$\begin{aligned} vF(x)\sum_i \phi_i(x)i &= \int_{\underline{a}}^{\bar{a}(x)} \sum_j m_j(s,x)ds \\ \Rightarrow vre^{-\tilde{q}(x)} &= \int_{\underline{a}}^{\bar{a}(x)} z(s)u(s,x)ds \end{aligned}$$

Returning now to the first order distribution in order to close the algorithm. Recall that $\tilde{H}(a,x)$ was the distribution of ability over links from whence the first order distribution was derived. This can now be defined as

$$\tilde{H}(a,x) = \frac{\int_{\underline{a}}^{a} z(s)u(s,x)ds}{\int_{\underline{a}}^{\bar{a}(x)} z(s)u(s,x)ds}$$

which, when plugged into the earlier derived expression for the first order distribution gives

$$\begin{aligned} H(a,x) &= e^{-r(1-\tilde{H}(a,x))e^{-\tilde{q}(x)}} \\ &= e^{-\frac{1}{v}\int_a^{\bar{a}(x)} z(s)u(s,x)ds} \end{aligned}$$

The pseudo probability that the ability of the best expected candidate of a firm is less than or equal to a is a negative exponential of the ratio of the mass of links of greater ability than a to the mass of firms remaining in the market.

Note that the term in the exponential has a similar flavor to the vacancy unemployment ratio from search and matching models, and in some sense captures the "tightness" of the market, modified to include the number of links that workers possess. In fact, were search behavior to be identical for all workers, such that $z(a) = z$, the term would be exactly

$$F(x)z\frac{\int_a^{\bar{a}(x)} u(s,x)ds}{F(x)v} = F(x)z\frac{U(x)}{V(x)}$$

The two objects thus identified, H(a,x) and u(a,x), sufficiently describe the execution of the matching algorithm and the expectations of workers and firms respectively about their prospects in the search-and-matching market. Differentiating these two objects with respect to a and x respectively yields the following system of partial differential equations

$$\begin{aligned} H_a(a,x) &= \frac{z(a)}{v}H(a,x)u(a,x) \\ u_x(a,x) &= \frac{z(a)v(x)}{v}H(a,x)u(a,x) \end{aligned} \tag{1.1}$$

with boundary condition for H given by $H(\bar{a}(x), x) = 1$ and the boundary condition for u to be given later in the equilibrium section.

Proposition 1.3.7. *The matching algorithm is pairwise stable*

Proof. Assume a putative, preferred bilateral deviation by x, a. x could have chosen a with acceptance but did not. $\Rightarrow\!\Leftarrow$ $\square$

1.4 Existence, Uniqueness and Topology of Equilibrium

The matching algorithm describes the microstructure of the matching market, and this section will incorporate that microstructure into a dynamic, general equilibrium framework. The equilibrium will exhibit a block structure, and its existence and uniqueness will be established in a manner corresponding to that block structure.

1.4.1 Workers

Workers can be either employed or unemployed. If employed, a worker is paid their contracted wage until the match is separated and they rejoin the pool of unemployed, implying a value function for a contracted worker of

$$V_t(a,w) = w + \beta\mathbb{E}\left[(1-\delta)V_{t+1}(a,w) + \delta U_{t+1}(a)\right]$$

An unemployed worker engages in home production in the beginning of a period. They can then engage in costly search, hoping to secure a job for the following period but must pay a cost given by $\xi(z)$ in order to search with intensity z. As seen in the matching algorithm section, the mass of workers that remains after all firms above x have moved is given by $u(a,x)$, then $n(a,x) = u(a,x)/u_t(a)$ is the probability that a worker of ability a remains unemployed. The worker's problem can then be stated as

$$U_t(a) = b - \xi(z) + \max_{z,x} \beta\mathbb{E}\left[\int_x^{\bar{x}(a)} V_{t+1}(a,w(t))d[n(a,t)] + n(a,F(x))U_{t+1}(a)\right]$$

The optimal policy for a reservation level for a worker is given by

$$\underline{x}(a) = \inf\{x : V_{t+1}(a,w(x)) \geq U_{t+1}(a)\}$$

which implies a worker will accept a wage up until they are indifferent between accepting that wage and returning to unemployment.

For $z(a)$, the policy rule is given implicitly by

$$-z(a)\beta \int_{\underline{x}(a)}^{\bar{x}(a)} V_{t+1}(a, w(t)) d[n(a,t) \ln n(a,t)] = \xi'(z(a))$$

. Here the worker trades off the cost of engaging in extra search relative to the improvement in their employment prospects, given the behavior of all other agents.

In any steady state equilibrium, $V_t = V, \quad U_t = U \quad \forall t$, and these reduce to

$$\begin{aligned} V(a,w) &= \frac{1}{\tilde{\beta}}\left[w + \beta\delta U(a)\right] \\ \Rightarrow U(a) &= \frac{1}{\tilde{\beta}}\left[1/\beta + \delta - n(a, \underline{x}(a))\right]^{-1} \left[\frac{\tilde{\beta}}{\beta} b - \xi(z) + \int_{F(R)}^{\bar{F}(a)} w(t) d[n(a,t)]\right] \end{aligned}$$

while the policy rules are

$$\begin{aligned} \underline{x} : \quad & \underline{x}(a) = \inf\{x : w(x) \geq (1-\beta)U(a)\} \\ z : \quad & -z(a)\beta \int_{\underline{x}(a)}^{\bar{x}(a)} w(t) d[n(a,t) \ln n(a,t)] = \frac{\tilde{\beta}}{\beta}\xi'(z(a)) \end{aligned}$$

1.4.2 Firms

The program of a firm can now be restated from the matching algorithm section in terms of efficiencies. A firm of type x solves the objective function given by:

$$J(x) = \max_{a,\tilde{x}} \beta \left[\int_a^{\bar{a}(x)} \frac{y(x,s) - w(\tilde{x})}{1 - \beta(1-\delta)} dH(s,\tilde{x}) + H(a,\tilde{x})(1-\delta_f) J_{t+1}(x)\right]$$

. A firm chooses their reservation ability a and which type they would like to try to mimic in the queue, $\tilde{x}$. Of course in the increasing wage equilibria sought here, it will be the case that $\tilde{x} = x$. Where the wage distribution is differentiable, the condition governing the optimal wage policy of firms–after steady state conditions have been imposed and a healthy serving of algebra–is given by:

$$w'(x) = -\frac{\int_{\underline{a}(x)}^{\bar{a}(x)} y_a(x,s) H_x(s,x) ds}{1 - H(\underline{a}(x), x)} \tag{1.2}$$

H_x is the derivative of H with respect to firm type x (and queue position modulo a constant), and is therefore an expression of how the distribution of best-available workers improves as one moves up the queue. Ergo, it captures in the wage-equilibrium condition the benefits of posting a higher wage in terms of the improvement of the talent pool a firm can recruit from. Differentiating $H(a, x)$ yields H_x as:

$$H_x(a,x) = -\left[\frac{z(\bar{a}(x))}{v} u(\bar{a}(x), x)\bar{a}'(x) - \frac{1}{v}\int_a^{\bar{a}(x)} z(s) u_x(s,x) ds\right] H(a,x)$$

There are two sources of improvement in the talent pool present in this derivative. The first term in brackets represents the improvement in the upper bound of the support of the distribution. When firms increase their wages, they increase the best quality worker that would be willing to accept them as an employer. Call this effect the *acceptance* effect.

The second term in brackets represents the marginal improvement in the quality of remaining workers in the pool when moving up the queue–workers of type a are flowing out of the market at rate $u_x(a, x)$. Call this effect the *competition* effect.

The firm, then, chooses their queue position to optimally trade off the benefit of moving up the queue–given by the expectation of the improvement in the marginal product of workers recruitable by the firm from both the acceptance and competition effect–against the costs of doing so, represented simply by the wage.

To complete the firm's wage policy, a boundary condition needs to be imposed on the wage differential equation. This can be done in one of two ways. Either a minimum wage is imposed $\underline{w}$, or a condition requiring that the lowest paying firm has no incentive to deviate downward in wage. I will assume for now a binding minimum wage policy $\underline{w}$ posted by the lowest ranked firm.

The reservation policy of the firm is straightforward, given by

$$y(x, \bar{a}(x)) - w(x) = \tilde{\beta}(1 - \delta_f)J(x)$$

. The firm is willing to accept a worker up until the point where the gains from waiting and searching again are larger than employing that worker.

1.4.3 Equilibrium

The last element of the model that needs to be pinned down prior to defining equilibrium is the transition of states between periods.

The remaining mass of unemployed workers of type a after the matching market is concluded is given by $u(a, \underline{x}(a))$, while $(u(a, \bar{x}(a)) - u(a, \underline{x}(a))$ have transitioned to employment. Obversely, $(\omega(a) - u(a, \bar{x}(a))$ is the mass of ability a that starts a period employed, δ of which lose their job and enter the unemployment pool for the next period. (Recall that ω describes the distribution over talent in the unit population of workers) Therefore, the steady state condition governing worker flows is given by

$$u_{ss}(a) = u(a, \bar{x}) = \frac{\delta\omega(a) + u(a, \underline{x})}{1 + \delta}$$

For firms, an inflow of $\bar{v}$ new vacancies enter the matching market in any period, distributed over efficiencies by the function $\lambda(x)$. $1 - H(\underline{a}(x), x)$ of these firms fail to find a match, of which δ_f are destroyed. Therefore the steady state of the law of motion for firm measures is given by

$$v(x) = \frac{\bar{v}\lambda(x)}{1 - H(\underline{a}(x))}$$

.

An Equilibrium for the model can now be defined.

Definition 1.4.1. *A stationary equilibrium of the model is defined as: Value functions* J, U, V*; policy functions* $\underline{x}(a), z(a)$ *for the workers and* $w(x), \underline{a}(x)$ *for the firms; measures* $H(a,x), u(a,x), v(x)$ *governing the populations of agents such that*

1. $w(x), \underline{a}(x)$ *solves the firm's problem given distributions and policy rules*

2. $\underline{x}(a), z(a)$ *solve the worker's problem given distributions and policy rules*

3. $H(a,x)$*,* $u(a,x)$ *solve (1.1), given* $\underline{a}(x)$*,* $\underline{x}(a)$*,* $z(a)$*, with boundary conditions consistent with functions* $u_{ss}(a)$*,* $v(x)$

1.4.4 Existence, Uniqueness and Topology

The structure of the equilibrium will exhibit a block-like nature–with discontinuities or non-differentiabilites being propagated downward throughout the matching market.

To begin, a condition describing the local differential equation of worker reservation policy to match that of the firm needs to be derived. Differentiating the worker's policy function gives

$$(\frac{1}{\beta} + \delta - n(a, \underline{x}))w'(\underline{x}(a))\underline{x}'(a) = -\int_{\underline{x}(a)}^{\bar{x}(a)} w'(t)n_a(a,t)dt$$

The worker's reservation rule $\underline{x}(a)$ reflects the relationship between the improving prospects of a worker as ability increases and the change in their reservation wage.

$n_a(a,t)$ represents how a worker's prospects improve in the matching market as their ability improves. Differentiating $n(a,x)$ gives

$$n_a(a,x) = \left[-\frac{z(a)v(\bar{x}(a))}{v}H(a,\bar{x}(a))\bar{x}'(a) - \frac{z(a)}{v}\int_{x}^{\bar{x}(a)} v(t)H_a(a,t)dt\right]n(a,x)$$

. Similar to the firm's rule, there are two effects present. The first term in brackets gives the acceptance effect for the worker. As ability improves, the highest efficiency firm that would be willing to hire the worker improves by $\bar{x}'(a)$, while a worker's chance of being the best candidate for that firm is $H(a, \bar{x}(a))$. The second term represents the competition effect for the worker, as their ability improves they marginally improve their prospects over $H_a(a, t)$ measure of candidates. Note the above does not contain the marginal effect from on $z(a)$ as this cancels with the cost of that search in the worker's problem.

The above differential equation, when paired with the firm's similar reservation equation, will describe a system of ordinary differential equations for the acceptance regions of the agents in the model. However, as will be shown in the following section, the acceptance effects–being composed of the reservation rules of agents "above" the current state in the matching algorithm–will introduce non-differential points. Therefore the equilibrium existence will have to be proven piece-wise, which is done in the following section.

1.4.4.1 Existence and Topology

Define finite subdivisions of the firm efficiency support and worker ability support in the following way. Initialize via

$$x_{N_x} = \bar{x}$$

$$a_{N_a} = \bar{a}$$

and

$$x_{N_x - 1} = \underline{x}(\bar{a})$$

$$a_{N_a - 1} = \underline{a}(\bar{x})$$

. Then define the subdivision iteratively via:

$$x_i = \max\{\underline{x}(\underline{a}(x_{i+2})), \underline{x}\}$$

$$a_i = \max\{\underline{a}(\underline{x}(a_{i+2})), \underline{a}\}$$

Consider the highest ability worker $\bar{a}$. This worker will choose a reservation firm $\underline{x}(\bar{a}) < \bar{x}$. Now consider the interval $I_{N_x-1} = [x_{N_{x-1}}, x_{N_x}]$. Within this interval, firms can feasibly hire from any of the available population given a connection, as $\bar{a} = \bar{a}(x)$. Presupposing a boundary condition at $\underline{w}_{N_x} = \underline{x}(\bar{a})$, the following proposition holds

Proposition 1.4.2. *Given $\underline{w}_{N_x}$ a unique solution to the wage equation exists on I_{N_x}.*

Proof. By the Picard-Lindelöf Theorem ☐

Next, consider a second interval in the efficiency support of firms given by $I_{N_x-2} = [x_{N_x-2}, x_{N_x-1}]$. As stated earlier, the policy function $\underline{x}(a)$ is increasing, therefore $\bar{a}(x)$ is increasing in equilibrium. Employ a change of variables to invert the condition describing worker reservation values as

$$\bar{a}'(x) = \frac{(1/\beta + \delta - n(\bar{a}(x), x))w'(x)}{-\int_x^{\bar{x}(\bar{a}(x))} w'(t) n_a(\bar{a}(x), t) dt} \tag{1.3}$$

Let x_{N_x-2} be given by $\max\{\underline{x}, \{x : \bar{a}(x) = a_{N_a-1}\}\}$, and suppose a boundary condition given by $\bar{w}_{N_x-2}$, then the following proposition holds:

Proposition 1.4.3. *Given $\bar{w}_{N_x-2}$ and w on subdomain I_{N_x-1} a unique solution $(w(x), \bar{a})$ exists that solves the ODE system given by 1.2 and 1.3*

Proof. By the Picard-Lindelöf Theorem ☐

.

While the above proposition might seem to imply that a continuous policy function for wages is assured on the interval I_{N_x-2}, this turns out to not necessarily be the case.

It remains to be shown that the solution to the ODE system is in fact increasing in both variables, as was assumed at the outset. The following proposition details the necessary and sufficient condition for the equilibrium wage function to be continuous at x^*

Proposition 1.4.4. *The equilibrium policies are continuous at* $x_{N_x-1} = x^* \iff$

$$\int_{x^*}^{\bar{x}} w'(t)(n(\bar{a},t)(1-n(\bar{a},t))dt > (1/\beta + \delta - n(\bar{a},x^*))n(\bar{a},x^*)\frac{\int_{\underline{a}(x^*)}^{\bar{a}(x^*)} y_a(s,x)H(s,x)ds}{1-H(\underline{a}(x^*),x^*)}$$

Proof: See appendix.

To see when this will obtain, rewrite the coupled ODE system as

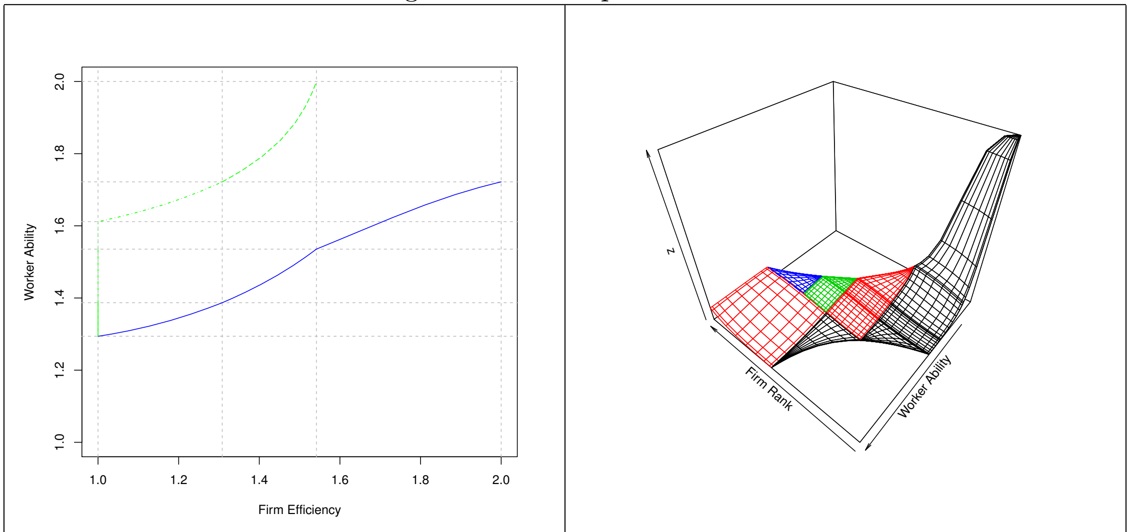
$$J(\bar{a},w)\begin{pmatrix} w' \\ \bar{a}' \end{pmatrix} = G(\bar{a},w)$$

then, w' and $\bar{a}'$ are positive iff the above proposition holds, which is merely a condition on the determinant of $J(\bar{a},w)$.

The intuition for the proof is the following. At x^*, the acceptance effect begins to be in effect for firms–the best candidate they can hire begins to degrade. This gives an increased incentive, in equilibrium, for wages to fall as the queue progresses. However falling wages feed back into the acceptance effect, such that w', $\bar{a}'$ are determined by the ODE system. If the feedback effects are too strong, it can be impossible for an infinitesimal change in both wages and $\bar{a}'$ to be consistent with equilibrium, so that a discontinuity in w must occur.

What a discontinuity in $w(x)$ implies is that the wage pdf, $f(w)$ has a gap in it where $w(x)$ is discontinuous. The implication is that at these points, workers are sorted into classes–or tiers–with stark differences in wages between them. We will see in the computational section that these equilibria are actually quite common for reasonable calibrations of the model, and that sorting into tiers is generally the norm. That this granularity emerges from an otherwise continuous model is somewhat surprising.

Figure 1.2: Example Solution



Acceptance Regions

u(a,x) on Acceptance Domain

1.5 Computation

This section describes the computational methods used to find an approximate solution to the equilibrium conditions. As noted in the section on the topology of the equilibrium, the $\{a_i\}$ and $\{x_j\}$ points represent non-differentiable points in the model. This fact, combined with the need to impose boundary conditions in a dynamic general equilibrium framework, necessitate a global solution method that can account for multiple subdomains. Pseudospectral element methods are an ideal choice because of their amendability so such problems, and because of their ease of numerical integration.

To give a sense of what kind of computational problem the equilibrium approximation represents, Figure 1.2 plots an example acceptance region and surface plot for the $u(a, x)$ function. The grey lines in the acceptance domain represent the non-differentiable loci that in a sense carve up the domain endogenously. These lines, along with the boundaries represented by the acceptance functions $\underline{x}$ and $\underline{a}$, will represent the boundaries of the mesh upon which the surface functions are computed.

1.5.1 Pseudospectral Element Methods

The structure of the equilibrium–in which multiple kinks and discontinuities will be reflected down the algorithm–requires a flexible computational approach to approximate. This section will detail such an approach. It relies on pseudospectral element methods to approximate both the policy functions and the surfaces of the distribution functions. The subdomains will be determined endogenously according to the mapping of nondifferentiable points down the matching queue. Thus, this section might be of interest to those looking to approximate diffeo-algebraic equations on a flexible grid.

Pseudospectral methods involve approximating a function by assuming a projection onto a functional basis. In this paper, I will be employing the basis of Chebyshev polynomials. I will give a brief overview of the details of this method, but refer the interested reader to Judd (1998) for a cursory introduction for economists, or Kopriva (2009) for a more thorough coverage of the methods used here.

Chebyshev approximation involves approximating a function via

$$f(x) = \sum_i c_i \psi_i(x)$$

where ψ_i are the Chebyshev polynomials of the first kind and c_i are the weights on those polynomials. Chebyshev polynomials have nice properties of orthogonality, meaning, among other things, that they provide a good approximating basis.

For the decision rules, the model requires approximating $N_a - 1$ policy rules for the workers, on the intervals defined by the partitioning algorithm that gives $\{a_i\}$. Similarly, the firm's decision rules will be demarcated along a grid defined by the $N_x - 1$ intervals $[x_i, x_i + 1]$. Because Chebyshev approximations need to be fit on the unit interval, this suggests the following approximation scheme for the decision rules:

$$\underset{\sim}{x}(a) = \gamma(q(a)) = \sum_i c_{gi}\psi_i(q(a))$$
$$q(a) = \frac{a - a_i}{a_{i+1} - a_i}$$

where the c_{gi} determine the weights and $q(a)$ maps to the unit interval. $z(a)$ can be defined similarly.

Firms' rules are approximated in a similar way on a given interval, with

$$w(x) = v(r(x)) = \sum_i c_{wi}\psi_i(r(x))$$
$$r(x) = \frac{x - x_i}{x_{i+1} - x_i}$$

The surface distributions are also approximated with chebyshev polynomials. This is accomplished via tensors of the single dimension polynomials, e.g.

$$f(x, y) = \sum_i \sum_j c_{ij}\psi_i(x)\psi_j(y)$$

.

The mapping of the surfaces to squares, however, is slightly more complicated than with the decision rules, because the domain on which they are defined is sometimes bounded by the rules themselves–leading to endogenously-defined non-square surfaces. In order to map the subdomains for these functions to the square, two types of mappings and their relevant jacobians will be used. A vertical stretch mapping will map the bottom and top boundaries–defined by curves–to the bottom and top of the square; while the horizontal stretch will do the same for the right and left boundaries. In the case where the bounded subdomain is three-sided, either mapping can be used, with a mapping between the corner singularity and a new side. Methods exist to make the mapping of triangle-like subdomains more efficient, (see Taylor et al. (2001) for an overview) however their implementation appeared to have little effect.

First define the mapping to a square for a rectangular surface defined by $[a_i, a_{i+1}] \times [x_j, x_{j+1}]$ via

$$q(a,x) = \frac{a - a_i}{a_{i+1} - a_i} \quad r(a,x) = \frac{x - x_j}{x_{j+1} - x_j}$$

Now, suppose a surface is bounded by the rules $\underline{a}(x)$ along its lower boundary and $\bar{a}(x)$ with $x \in [x_j, x_{j+1}]$ along its top boundary. Define the vertical stretch mapping as

$$q(a,x) = \frac{a - \underline{a}(x)}{\bar{a}(x) - \underline{a}(x)} \quad r(a,x) = \frac{x - x_j}{x_{j+1} - x_j}$$

. This will map the subdomain on which the surface is defined onto the square by stretching it vertically according to the decision rules that bound it.

Alternatively, invert the bounding functions for $\underline{x}(a)$ and $\bar{x}(a)$ with $a \in [a_i, a_{i+1}]$. Define the horizontal stretch mapping as

$$q(a,x) = \frac{a - a_i}{a_{i+1} - a_i} \quad r(a,x) = \frac{x - \underline{x}(a)}{\bar{x}(a) - \underline{x}(a)}$$

. This will map the subdomain on which the surface is defined onto the square by instead stretching it horizontally. All three of these mappings will be used to approximate the surfaces depending on how it is bounded and which decision rules are being updated and therefore how the surfaces need be numerically integrated.

Given an appropriate mapping to the square, the surface functions can then be approximated as as

$$H(a,x) = B(q(a,x), r(a,x)) = \sum_i \sum_j c_{bij} \psi_i(q) \psi_j(r)$$

$$u(a,x) = D(q(a,x), r(a,x)) = \sum_i \sum_j c_{dij} \psi_i(q) \psi_j(r)$$

.

Obtaining approximations is then a matter of defining an appropriate concept of a residual and minimizing that residual according to a chosen method. In this paper I will minimize the residual at collocation points given by the Gauss-Chebyshev points of the interval or surface in question.

The residuals for the surface functions are given by the PDE system that resulted from the matching algorithm, given the decision rules of the firms and workers. For the vertical stretch mapping, these are given by

$$\begin{aligned} G_D \equiv & D_r(q,r)\frac{1}{\Delta_x} - D_q(q,r)\left(\frac{q\Delta_a}{\gamma'(q(\bar{a}(x)))(\bar{a}(x)-\underline{a}(x))} + \frac{(1-q)\alpha'(r)}{\delta x(\bar{a}(x)-\underline{a}(x)}\right) \\ & - \frac{z(a(c,r))v(r)}{v}D(q,r)B(q,r) \\ G_B \equiv & B_c(q,r)\frac{1}{\bar{a}(x)-\underline{a}(x)} - \frac{z(a(c,r))v(r)}{v}D(q,r)B(q,r) \end{aligned}$$

where subscripts denote derivatives wrt that variable. These residuals come from the required change of variables to map the surface PDE to the square. The horizontal stretch mappings are given similarly as

$$\begin{aligned} G_D &\equiv D_r - \frac{z(a(c,r))v(r)}{v}DB(\bar{x}(a)-\underline{x}(a)) \\ G_B &\equiv B_c\frac{1}{\Delta_a} - B_r\left(\frac{r\Delta_x}{\alpha'(r(\bar{x}(a)))(\bar{x}(a)-\underline{x}(a))} + \frac{(1-r)\gamma'(c)}{\Delta_a(\bar{x}(a)-\underline{x}(a))}\right) - \frac{z(a(c,r))v(r)}{v}DB \end{aligned}$$

.

1.5.1.1 Collocation

Approximation will be obtained by miniziming the equilibrium residuals at collocation points given by N_{c+1} Gauss-Chebyshev points. These points are defined as

$$x_j = \cos\frac{(j-1/2)\pi}{N} \quad 0 \le j \le N_c$$

where N_c is chosen to provide an adequate level of accuracy and represents the order of the highest level of Chebyshev polynomial used. The Gauss-Chebyshev points represent the roots of this highest order polynomial, and minimizing at these points yields a nonlinear system of N_{c+1} residual equations and N_{c+1} unknown weights $\{c_i\}$ for the univariate functions, and N_{c+1}^2 residual equations and N_{c+1}^2 unknown weights $\{c_{ij}\}$ for the bivariate surface equations.

While methods exist to collocate on sparser grids, e.g. Judd et al. (2014), there are two reasons not to do so here. First is that numerical integration for the decision rules will occur along the lines defined by the tensor product of the subdomain, so the highest possible accuracy is ensured when the full tensor product is used. Second, as we will see, the factorizations of the matrices representing the chebyshev polynomials at the Gauss-Chebyshev points can be precomputed when the full tensor product is used–greatly increasing the speed of computation by removing the need to invert large matrices.

The matrix of $2N_{c+1}^2$ residual equations for the distribution surfaces can be written as

$$A_{dr} \circ D_r + A_{dc} \circ D_c + A_d \circ D \circ B$$

$$A_{br} \circ B_r + A_{bc} \circ B_c + A_b \circ D \circ B$$

where $\circ$ represents the Hadamard product and the A matrices are defined according to the appropriate mapping. Let Ψ represent the matrix of elements $\psi_{ij} = \psi_i(x_j)$ Chebyshev values at the Gauss-Chebyshev points and $\tilde{\Psi}$ their derivatives. Further, let C_D and C_B represent the coefficient matrices for D and B respectively, with typical element c_{ij}. Then the residual equations can be rewritten as

$$A_{dr} \circ \Psi' C_D \tilde{\Psi} + A_{dc} \circ \tilde{\Psi}' C_D \Psi + A_d \circ \Psi' C_D \Psi \circ \Psi' C_B \Psi$$

$$A_{br} \circ \Psi' C_B \tilde{\Psi} + A_{bc} \circ \tilde{\Psi}' C_B \Psi + A_b \circ \Psi' C_D \Psi \circ \Psi' C_B \Psi$$

These equations should be put into a vector format in order to run a Newton-Raphson method on them, and this will be done in a manner that makes the Jacobian easy to invert. Taking the transpose of the above equations, and applying the *vec* operation will yield

$$a_{dr} \circ \Psi' \otimes \tilde{\Psi}' c_d + a_{dc} \circ \tilde{\Psi}' \otimes \Psi' c_d + a_d \circ \Psi' \otimes \Psi' c_d \circ \Psi' \otimes \Psi' c_b$$

$$a_{br} \circ \Psi' \otimes \tilde{\Psi}' c_b + a_{bc} \circ \tilde{\Psi}' \otimes \Psi' c_b + a_b \circ \Psi' \otimes \Psi' c_d \circ \Psi' \otimes \Psi' c_b$$

where $\otimes$ is the kronecker product, $c_d = vec(C'_D)$ and $c_b = vec(C'_B)$. This uses the relation that $vec(ABC) = (C' \otimes A)B$. The mapping is now rendered into a vector of N_{c+1}^2 residuals. The Jacobian of this system can be written in block form as

$$\begin{pmatrix} A_{dr}\Psi' \otimes \tilde{\Psi}' + A_{dc}\tilde{\Psi}' \otimes \Psi' + A_{bd}\Psi' \otimes \Psi' & -A_{dd}\Psi' \otimes \Psi' \\ -A_{bb}\Psi' \otimes \Psi' & A_{br}\Psi' \otimes \tilde{\Psi}' + A_{bc}\tilde{\Psi}' \otimes \Psi' + A_{db}\Psi' \otimes \Psi' \end{pmatrix}$$

where now the coefficient matrices are $N_{c+1}^2 \times N_{c+1}^2$ diagonal matrices with $A_{db} = diag(vec(D) \circ a_d)$ etc.

The above Jacobian is $2N_{c+1}^2 \times 2N_{c+1}^2$, which can end up being a very large matrix for reasonable values of N_c. However, this is mitigated by some attractive properties of Chebyshev polynomials at Gauss-Chebyshev points. Ψ and $\tilde{\Psi}$ can both be decomposed via LQ factorization on the same orthogonal basis Q as $\Psi' = LQ$ and $\tilde{\Psi} = \tilde{L}Q$ where $L, \tilde{L}$ are lower triangular matrices. Further, the kronecker product implies $\Psi' \otimes \Psi = (L \otimes L)(Q \otimes Q)$. As shorthand, let $LL = L \otimes L$ and $QQ = Q \otimes Q$, then we can rewrite the Jacobian as

$$\begin{pmatrix} A_{dr}L\tilde{L} + A_{dc}\tilde{L}L + A_{bd}LL & -A_{dd}LL \\ -A_{bb}LL & A_{br}L\tilde{L} + A_{bc}\tilde{L}L + A_{db}LL \end{pmatrix} \begin{pmatrix} QQ & 0 \\ 0 & QQ \end{pmatrix}$$

. The matrix on the right is orthogonal and therefore easily invertible. The left matrix is block-lower-triangular. After accounting for the top right block via block inversion, it can be solved via forward propagation. Therefore, an expensive $O((2N_{c+1}^2)^3)$ computation need not be computed when solving the system of equations for a Newton-Raphson method.

1.5.1.2 Row Computation

The boundary conditions on the surface functions mean that the surface functions cannot be solved in isolation on their subdomains. Via the original PDE system, the boundary conditions imply that the H function must match its neighbors above and below along their shared boundaries, while the u function must match the left and right boundaries, with the added condition that the left and rightmost boundaries of any horizontal section, or "row", must accord with the steady state labor characterization.

This structure–where the left-right boundaries "wrap" around the surface imply that computing the surface functions iteratively by rows will be an ideal iterative method. At each row, the bottom boundary of B, or the ability first-order distribution, will be fed to the following row to serve as the top boundary.

This concludes the computation of the distribution surfaces. The next subsection will document how to update the worker and firm decision rules given the surface values.

1.5.1.3 Worker Updates

The worker's problem requires the calculation of several numerical integrals over the regions of their potential hiring. Luckily, approximations via Chebyshev polynomials provide an easy method for doing so at the collocation points by integrating the polynomials themselves. Restating the differential equation from the worker's problem:

$$(\frac{1}{\beta} + \delta - n(a, \underline{x}))w'(\underline{x}(a))\underline{x}'(a) = -\int_{\underline{x}(a)}^{\bar{x}(a)} w'(t)n_a(a,t)dt$$

. Suppose the distribution functions u and firm policy rules are given. The worker's problem will involve integration over M subdomains within their respective row, as defined in the section above. Index these M subdomains with j from left to right. Then the worker's condition can be rewritten at a Gauss-Chebyshev node q_i as

$$\gamma'(q_i) = -\frac{\sum_{j=1,M} \Delta_{xj}^2 \int_0^1 \nu_j'(\bar{r} + (\bar{r} - \underline{r})t)\frac{D_{cj}(q_i,t)}{D_M(q_i,1)}dt}{\left(\frac{1}{\beta} + \delta - \frac{D_1(q_i,0)}{D_M(q_i,1)}\right)\nu_1'(\underline{r}(q_i))}$$

. Inverting $\tilde{\Psi}'$ along with the boundary condition allows computation of the update c_g of the worker's acceptance region in this interval.

1.5.1.4 Firm Updates

The firm update requires computation of numerical integrals similar to those of the worker's problem. Restating the differential equation for wages from the optimal firm behavior:

$$w'(x) = -\frac{\int_{\underline{a}(x)}^{\bar{a}(x)} y_a(x,s)H_x(s,x)ds}{1 - H(\underline{a}(x), x)}$$

. In terms of Chebyshev polynomials, at a collocation point r_i this integral is split into M subdomains within a firm's respective column, to wit:

$$\nu'(r_i) = \frac{\sum_{j=1}^{M} \Delta_{Fj} \int_0^1 y_a(x(r_i), s)B_{rj}(s, r_i)ds}{1 - B_1(0, r_i)}$$

with the boundary condition $\nu(0) = \bar{\nu}$ provided by the column to the left. Inverting the Chebyshev derivative matrix $\tilde{\Psi}$ concatenated with the boundary condition gives an update of the c_ν weights. The remaining firm rules, α and υ are updated to match according to their respective equilibrium conditions.

Given the worker and firm update rules, the following algorithm gives the basic structure of the equilibrium computation. In an outer loop, the firm rules are updated to convergence. In an inner loop, taking as given the firm rules, the distributions are first computed taking as given all rules, then the worker rules is updated to convergence.

Algorithm 1: Computation of Equilibrium

Result: Equilibrium

Guess single firm rule ν_0;

Compute $\underline{x}(\bar{a})$ and split firm rule;

while $|c_{\nu t} - c_{\nu t-1}| > \varepsilon \quad \forall \quad \nu$ **do**

  Update all worker rules (from right to left) to convergence;

  Update $c_{\nu t+1}$ (from left to right);

  t = t+1;

end

. The resulting weights approximate the equilibrium objects of the model.

1.6 Comparative Statics and Numerical Results

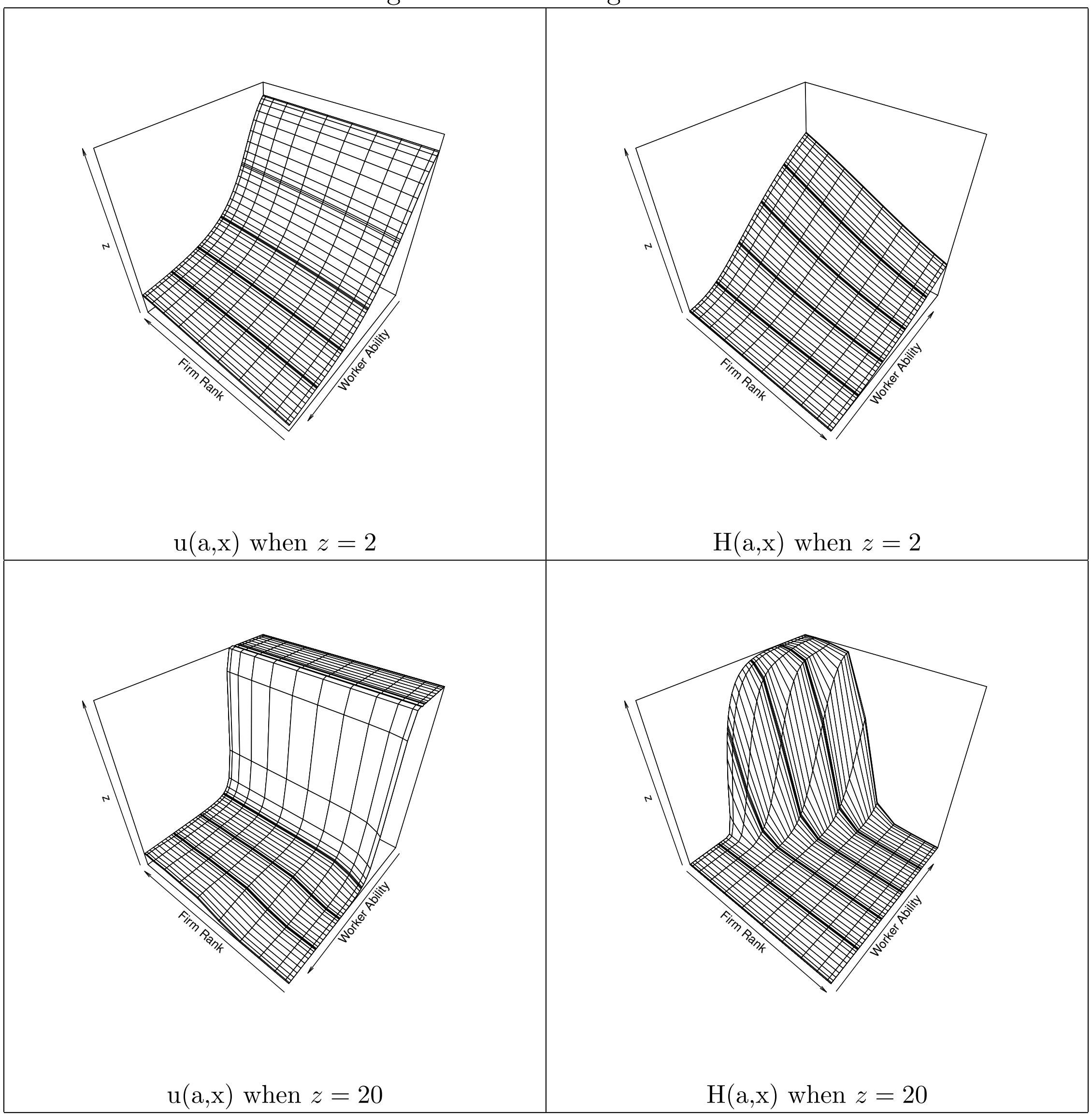
1.6.1 Matching Surface

In order to get a sense of the matching algorithm and especially to demonstrate the intuition for the exit of labor force market participants, it is useful to view the distribution surfaces on the square for alternative calibrations. Figure 1.3 shows the distribution surfaces for the lower half of the ability support for workers–one with a relatively low search value of $z = 2$ for all participants and one with a relatively high level of $z = 20$.

When $z = 20$, the model surfaces are relatively flat, until a great deal of curvature results around the line $a = \bar{a} - \frac{v}{U(\bar{x})}x$. This accords with intuition, and in the limit of perfect information, we would expect all hiring to occur exactly on this line.

When $z = 2$, however, the surface is more "forgiving", and the steepness–meaning the hiring–is spread across the domain. In this world, matching is relatively noisy, and firms and workers can match with participants that are of relatively larger difference in rank. What this implies for the very lowest ranked of workers, that is, below $\frac{v}{U(\bar{x})}\underline{x}$, is that they have a possibility of "benefiting from noise"–they can be hired because search frictions are

Figure 1.3: Matching Surfaces



prohibitive enough that they can on occasion be the best available worker. When $z = 20$ this is clearly no longer the case–the probability of any of these workers of being hired is essentially null, and they will have no incentive to remain in the market.

1.6.2 Comparative Statics

This section presents model statistics for various calibrations of the search costs. It will adopt the following perfectly inelastic functional form for search costs. Workers can pay an ε small value to participate in the market. If they do so, they receive the marketwide allocation of z random draws, should they not pay the cost they receive nothing. There will be a cutoff rule which will dictate that any worker above a^* will search and below will not.

1.6.2.1 Calibration

In order to calibrate the model I will take some parameter values from the literature and assign others agnostically. This calibration is meant to serve as a benchmark point in the parameter space in order to demonstrate the effects of the model more than to realistically reflect the conditions of the US labor market–a task I leave for future work.

The model period will be one quarter. $\beta = 0.99$ to match an interest rate of 4.1% per annum. δ is set to match the statistic found in Shimer (2012). δ_f is somewhat of an ignored parameter in the literature. It is briefly mentioned in Davis et al. (2013) where it is assumed that $\delta = \delta_f$. However such an assumption seems intuitively incorrect: a matched and producing job would likely have a much higher probability of surviving and remaining competitively viable than a vacancy that has failed to match with a candidate. For this reason I set δ_f to be a good deal higher than δ, such that an unmatched vacancy has a survival probability into the next period of 0.75.

The utility from unemployment is set agnostically at 0. It does not seem to have outside effect on the model behavior. The distributions of ability and efficiency are both assumed to be uniform with identical support. $\bar{v}$ is set at 0.55. This is to account for the fact that approximately half of all hires come from job-to-job transitions, meaning the vacancy-unemployment ratio overstates the vacancies available to the unemployed. $\underline{w}$ is set at 1 and $\sigma = 1$ to capture a reasonably strong complementarity between firm and worker ranks.

The following table presents the calibration of baseline parameters. These parameters will remain fixed as the model is solved over different calibrations for search costs.

Table 1.1: Baseline Model Parameters

Parameter	Value
β	0.99
δ	0.1
δ_f	0.25
b	0
$[\underline{a}, \bar{a}]$	[1,2]
$[\underline{x}, \bar{x}]$	[1,2]
$\bar{v}$	0.055
$\underline{w}$	1
σ	-1

1.6.2.2 Statics

Figure 1.4 displays the increasing wage inequality generated by the model when search frictions decline. As z increases from 3 to 16, wages at the bottom end of the distribution (10th and 25th % ile) actually experience a slight decline in wages. Firms at the low end of the efficiency distribution, and therefore who are targeting lower ability workers, see little reason to raise wages as they do not see increased competition from more information.

On the other hand, workers at the 75th and 90th percentile of ability experience large increases in wages as search frictions decline. Two effects are present here. Not only is the wage offer distribution steepening due to increased competition (see 1.5); but high ability workers are now more likely to be hired at these jobs, as the matching becomes more narrowly sorted. These two effects combine to create large wage jumps relative to lower ranked workers. Put bluntly, it is the most talented within the employment pool who reap almost all of the benefits of increased matching efficiency.

Figure 1.5 depicts the wage offer function and unemployment levels for the stationary model under two calibrations, $z = 4$ and $z = 16$. When information increases in the matching market, the highest ranked firms increase wages to defend against firms from below wishing to steal their preferred hire. When information is low, the probability of finding a preferred candidate, let alone having them stolen, is low; and so the incentive to

Figure 1.4: Wage Inequality

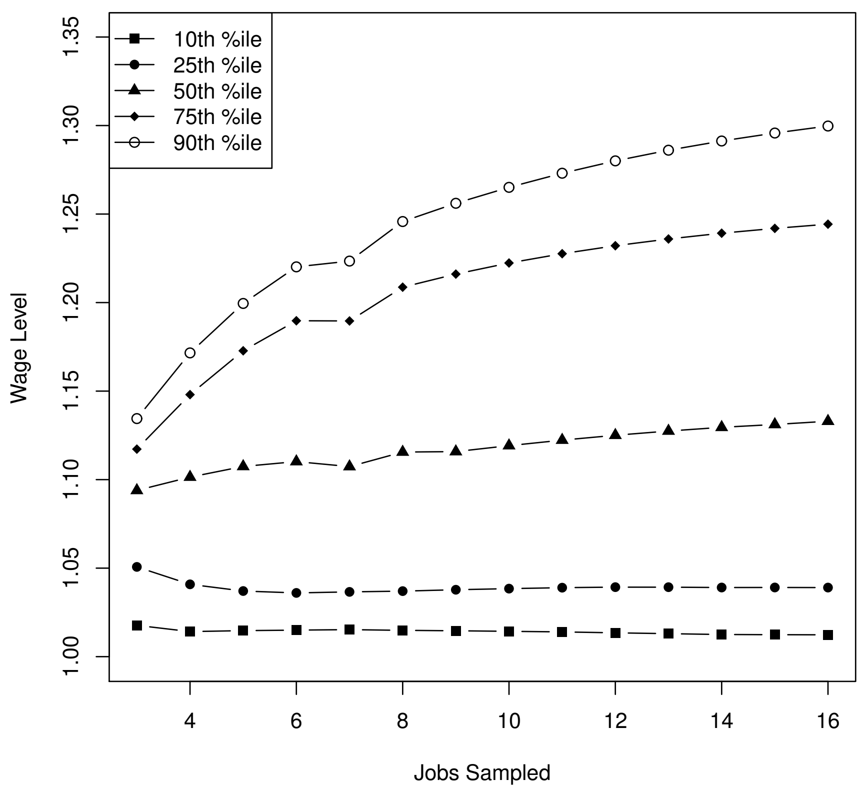
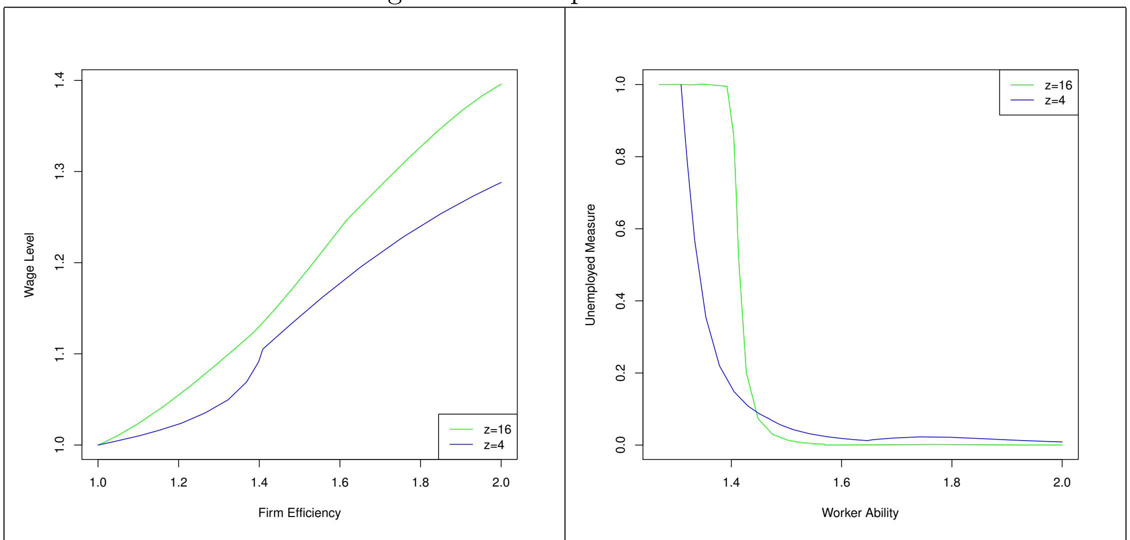


Figure 1.5: Comparative Statics



Wage Function Under z=4,16 | Unemployment Level under z=4,16

post high wages is lessened.

The second image depicts the same effect observed as on the matching surface. As information increases, the slice of the market that benefits from search noise–here the domain bounded by the upper kinks of the two lines–is squeezed out of the market. Participants of ability between roughly 1.35 and 1.4–representing about 5% of the market, lose their incentive to participate and so exit.

1.7 Empirics

This section presents empirical evidence for the prediction of the model regarding labor force participation: that labor force participation will decline in the presence of cheaper search. It will specify a logistic regression of individual labor force exit decisions from the CPS microdata. Identification of the effect of search efficiency of workers will be obtained through differentials in internet job search activity at an occupational and/or industry level.

The main prediction of the model is that an increase in search efficiency at a market level should serve to force out the lowest ranked workers–who no longer find it advantageous to try searching. Testing this prediction requires an econometric specification that can include information on heterogeneity in worker rank at an individual level, as well as the level of search efficiency at a market-wide level. Hence, let y_i be a log odds metric of an individual's likelihood of exiting the labor force, modeled as

$$y_i = \alpha + \beta \mathbf{x}_i + \Gamma s_{m(i)} * \mathbf{r}_i + \gamma s_{m(i)} + \Lambda \mathbf{z}_{m(i)} + \varepsilon_i$$

where $\mathbf{x}_i$ is a vector of demographic characteristics, $\mathbf{r}_i$ is a vector of individual level ranking metrics, $s_{m(i)}$ is the level of search efficiency within the market that i is searching, $\mathbf{z}_{m(i)}$ is a vector of market level controls and ε_i is i.i.d. noise. The intent will be to assess the coefficients of Γ, which describe the effect of an interaction between market-level search efficiency and individual level rankings on probability of labor force exit. Of course, individual rank is not observable in the data, and so I will rely on proxy variables that are plausibly highly correlated with the underlying rank of the worker: binary variables for young workers and uneducated workers.

The data for fitting the above specification will come from the CPS microdata, provided by IPUMS. These data involve surveys of two four-month panels of individuals with an 8-month gap in between. The limited panel dimension of the survey allows for tracking of labor force transitions between employment, unemployment, and out-of-labor-force, as well as individual-level information on worker characteristics.

The dependent variable in the econometric specification is the log odds of exiting the labor force. As stated, labor force exit can be constructed from the CPS microdata by tracking individuals' labor force status, and counting as an affirmative observation any transition from in the labor force (employed or unemployed) to out of the labor force. I run alternate specifications of the model that consider a sample of only those unemployed in the base year and also either employed or unemployed. Table 1.2 below shows a summary for the individual level variables within the data set, including the dependent variable of rates of labor force exit.

Table 1.2: Individual Level Variables

Statistic	N	Mean	St. Dev.	Min	Max
exit	4,449,303	0.030	0.169	0	1
age	4,449,303	41.505	10.955	22	60
hs dropout	4,449,303	0.071	0.257	0	1
no college	4,449,303	0.559	0.496	0	1
male	4,449,303	0.523	0.499	0	1
white	4,449,303	0.819	0.385	0	1
Mar. Stat.	4,449,303	0.597	0.491	0	1

In order to identify the effect of cheap search in the CPS data, I turn to a recently added Internet and Computer Use supplement. This supplement asks broad questions about computer use at home and at work, including a question about whether the respondent used the internet to search for a job within the past 6 months. Aggregating the proportion of responses of this internet search variable allows construction of the $s_{m(i)}$ variable from the econometric specification and captures the amount of cheap search within the marketplace that the worker is searching. Therefore, $s_{m(i)}$ represents the proportion of workers within a market who are using the internet to search for a job. Because this variable might proxy for an overall desire to search within the market (both on and off the job) due to slack conditions, I will run alternate specifications that restrict the sample to only unemployed workers, so $s_{m(i)}$ is the proportion of unemployed workers using internet search within a market.

I will run specifications that aggregate market level variables according to both a worker's industry and their occupation in the base year. While this sidesteps the issue of industry and occupation switching, observed rates of such switches are quite low, especially for occupation. The level of aggregation will be the 453 four-digit SOC codes for occupation–from the CPS, standardized to the 2010 codes by IPUMS–and 223 three-digit SIC codes for industry–standardized to 1990 values again by IPUMS. Below is a sample summary table of $s_{m(i)}$ as well as market-level control variables for the SOC occupation-based aggregation.

The results of fitting the various specifications are shown in Table 1.4. An m subscript

Table 1.3: Market Level Variables

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Urate	453	0.06	0.06	0.00	0.03	0.08	1.00
Wage	451	926.84	339.43	294.29	657.59	1,131.61	2,020.61
Wage^2	451	973,998	723,522	86,606	432,430	1,280,530	4,082,849
N empl.	454	21,123	170,619	45	1,350	10,681	3,402,685
isearch	386	0.68	0.27	0.00	0.54	0.88	1.00

denotes a market level variable. The last four rows denote the effects due to internet search at the market level, with the last three being the coefficients of interest for search interacted with low rank. The effect of the level of internet search overall is a negative one; that is, searchers in markets with high internet search are less likely to exit into unemployment. This could be due to two possible effects. The first is that it is representing an underlying level of technical sophistication in the market. Another possibility is that markets with high internet search are more efficient and workers of high ability are therefore more willing to stay in unemployment. The fact that the specification for only the unemployed does not have a significant effect suggests the former explanation (though it could be due to a sample size effect.)

All three interaction terms for search with ranking variables are positive, with significance in most specifications. Especially strong is the age interaction effect, which displays strong significance for every specification. Also suggestive is that the interaction for high school dropouts is stronger than for those with no college diploma. This implies that the crowding out effects are most salient for the very lowest-ranked candidates, and the effect dissipates as one moves up in education levels, as the model predicts. The overall takeaway is that higher rates of internet job search are associated with higher rates of labor for exit for *only* the young and uneducated workers in a market.

Table 1.4: Logistic Regressions

	Dependent variable:			
	Labor Force Exit			
	Model 1	Model 2	Model 3	Model 4
Constant	−2.381***	−0.466***	−2.340***	−0.331***
	(0.045)	(0.092)	(0.052)	(0.104)
age	−0.012***	−0.004***	−0.012***	−0.004***
	(0.0004)	(0.001)	(0.0004)	(0.001)
hs dropout	0.564***	0.180*	1.015***	0.307***
	(0.052)	(0.099)	(0.050)	(0.095)
no college	0.195***	0.113	0.388***	0.171**
	(0.039)	(0.082)	(0.039)	(0.080)
male	−0.538***	−0.409***	−0.570***	−0.420***
	(0.007)	(0.012)	(0.006)	(0.012)
white	−0.376***	−0.184***	−0.418***	−0.196***
	(0.007)	(0.012)	(0.007)	(0.012)
married	−0.189***	0.001	−0.221***	−0.005
	(0.006)	(0.011)	(0.006)	(0.011)
Urate_m	7.146***	−0.272	7.182***	−1.362***
	(0.109)	(0.190)	(0.128)	(0.231)
Wage_m	−0.001***	−0.001***	−0.001***	−0.001***
	(0.00005)	(0.0001)	(0.0001)	(0.0001)
Wage_m^2	0.00000***	0.00000***	0.00000***	0.00000***
	(0.00000)	(0.00000)	(0.00000)	(0.00000)
N firms_m	0.00000***	0.00000***	0.00000***	−0.00000***
	(0.00000)	(0.00000)	(0.00000)	(0.00000)
isearch	−0.812***	−0.316	−0.846***	−0.107
	(0.107)	(0.229)	(0.113)	(0.237)
isearch×(age< 30)	0.575***	0.221***	0.673***	0.260***
	(0.029)	(0.054)	(0.029)	(0.054)
isearch×(hs dropout)	0.447***	0.760**	−0.139	0.667**
	(0.166)	(0.312)	(0.161)	(0.297)
isearch×(no college)	0.183	0.378	0.166	0.423*
	(0.124)	(0.254)	(0.122)	(0.248)
Include Employed?	Yes	No	Yes	No
Observations	4,407,027	236,758	4,407,110	236,759
Log Likelihood	−552,859.900	−112,509.300	−555,249.700	−112,582.100
Akaike Inf. Crit.	1,105,750.000	225,048.700	1,110,529.000	225,194.200
Note:				*p<0.1; **p<0.05; ***p<0.01

1.8 Conclusion

This paper constructed a model of simultaneous, random search to study the effects of cheaper search on employment matching markets.

An environment of endogenous search effort was described that allowed workers to sample multiple firms and an algorithm describing pairwise stable matching was provided. Further, the computation of the model equilibrium via pseudospectral element methods was demonstrated. As the matching algorithm tightens around the locus representing the vacancy-unemployment ratio, those workers who are the least-preferred lose their incentive to participate in the matching market as they can no longer "get lucky" and match with a firm that itself got unlucky.

Given the structure of the matching market, firms and workers make decisions about their acceptance sets, and firms specifically choose what wage to post. The endogenous wage distribution thus induced generates increasing wage inequality in response to cheaper search, as competition for workers generally builds from below and peaks for the highest-ability workers—put simply, in a high information environment, high-ranked firms know they will meet a high ranked worker, and they know they will lose that worker to competitors if they try to post a low wage.

There are several avenues for further research, two of which I will explicitly mention here. First, the model lacks on the job search. Because close to half of all labor market transitions involve job-switching, this will be a key element to consider in future work.

An interesting extension of this modeling environment involves both the education decisions of workers, as well as the job-training behavior of firms. With regard to the former, if education–which can differentiate a candidate in an absolute sense from another–serves to move a candidate up the ability ladder, the premium of education should increase. As for on-the-job training, because firms with more applicants can more readily match with a better fit, their need to train a worker to a desired level post-hire is lessened. We should then expect to see less “skilling up” by workers on the job–an erstwhile valuable source of education for workers.

Chapter 2

Labor Reallocation and Sectoral Shocks

2.1 Introduction

Real business cycle theory relies on the concept of a "shock" to generate fluctuations in an economy. An example of an aggregate shock is monetary policy—which affects the costs of borrowing money for every agent in an economy. Sectoral shocks affect predominately the production in one sector. For example, the shock might be productive like the invention of a new sector-specific technology, or disruptive, like a spike in the cost of a certain raw material. The existence of these two kinds of shocks begs the question of which is more important for the business cycle: do shocks to individual sectors aggregate to drive output movements; or are these shocks washed out in the aggregate, leaving common shocks as the only true culprit in business cycle fluctuations? This paper will assess the contribution of aggregate vs. sectoral shocks by building a real business cycle model calibrated to match realistic structure in the market for all three major inputs to production: material inputs, capital goods and labor.

The literature that structurally addresses the contribution of sector-specific vs. aggregate shocks dates to Long and Plosser (1987), who build a model that incorporates commodity inputs in production and argue these linkages can generate significant aggregate fluctuations. This literature blossomed with the work of Horvath (2000) and Dupor (1999), who stake out opposite sides of a debate—with Horvath arguing for the importance of sectoral shocks under one set of assumptions and Dupor demonstrating their irrelevance under a different set.

Sector-specific shocks were revisited more recently with Acemoglu et al. (2012), Foerster et al. (2011) (henceforth FSW) and Atalay (2017). The former paper interprets material purchases between firms as a network. It gives theoretical justification for the importance of sectoral shocks in the presence of asymmetry in the material network. The precence of "star suppliers"—and their higher-order propagating effects—serve to elevate sectoral shocks to aggregate importance.

Atalay and FSW take a more empirical approach in attempting to estimate precisely how much aggregate fluctuation is generated by sectoral shocks. FSW employs a structural model of intersectoral material and capital linkages to filter out their comovement-generating effects. These shocks are decomposed with factor analytic methods to assess the importance of sector-specific vs. aggregate shocks. Their findings indicate that prior to the great moderation (pre-1984), aggregate shocks account for a majority (approx 85%) of output volatility while sectoral shocks account for roughly half of all volatility after 1984.

Atalay innovates on FSW by incorporating non-unitary elasticities into production. The paper estimates production elastiticies in materials and final goods that are siginficantly below 1 by measuring the effect of price movements on input shares. When elasticities are low—as found in that paper—the role of sectoral shocks is amplified, as firms are less free to substitute between inputs in the face of disturbances. When realistic elasticities of substitution are incorporated, sectoral shocks are found to contribute well over half of all aggregate variation in Atalay's chosen measure. For the estimated elastiticies in that paper, almost all volatility is conferred to sectoral shocks. FSW and Atalay are the closest two papers to the current one. However, in both of these papers, labor is treated as a stock variable that is freely recruited in a spot market during every period.

In this paper, I will demonstrate that the re-allocation of labor between sectors occurs according to a definite network structure: meaning the sector in which a transitioning worker is located is important for predicting the sector into which they will move. Motivated by this network structure, I incorporate a Lucas-style frictional labor market into a model with material and capital linkages as in Atalay. In this model, industries are located on islands, and workers face an island-dependent probability of successfully transition to another island. So, for example, the probability that a worker will succeed in an attempted transition between the "Construction" sector and "Oil and Gas Extraction" is different than between "Construction" and "Education". These probabilities allow me to calibrate the model so that it matches the observed network structure of labor-reallocation.

I find that incorporating a frictional labor market network revises upward the estimate of the contribution of aggregate shocks from the numbers found in Atalay. For most calibrations, the contribution of these shocks is around 50% of aggregate output volatility, even with the low production elasiticites found in Atalay. This is due to two potential effects: a general intertemporal smoothing effect; and an idiosyncratic network effect.

The general effect is that all firms face frictions in adjusting their labor in response to shocks. The implication for shock propagation is to remove some of the bite of the material linkages: when a positive shock is propagated to a given sector, that sector faces an intratemporal labor supply curve that is much steeper than when labor is homogeneous and allocated within a period in a spot market; conversely, when a negative shock hits, the effect is somewhat ameliorated by an abundance of cheap labor before workers can re-allocate.

The network effect works by a similar mechanism, but it greatly reduces the number of "channels" through which workers can re-allocate, meaning that re-allocation can be prolonged, inefficient, and might amplify the propagation-numbing effects of frictional labor.

However, I find that incorporating the network of re-allocation probabilities so as to match the observed labor re-allocation measure does little to change my estimate of sectoral vs. aggregate shocks or the comovement of sectoral output. The interpretation is that despite the structure and sparsity of labor re-allocation between industries, the network is dense enough to robustly re-allocate workers in a way that does not propagate shocks differently than a fully connected network.

This paper also relates to a large literature on sectoral labor reallocation. Dvorkin (2013) incorporates a Lucas island model into a real business cycle model with material linkages and finds that labor frictions are important for the co-movement of business cycle variables. Pilossoph (2014) builds a similar model to demonstrate that intersectoral labor frictions do not generate large levels of unemployment in a two-sector setting. The current paper similarly finds that the structure in which cyclical upgrading occurs is relatively unimportant for the propagation of sectoral shocks; but that general frictions to re-allocation do have large implications for the source of business cycle fluctuations.

2.2 Data

This section details the various data sources that will be used in either calibrating the model or as the productivity series from whence the underlying disturbances are filtered. The first source of data is sectoral output data from the BEA, which will be the observable quantity used to assess industry-level production. The next three data sources will each be used to respectively calibrate one of the three main inputs to production in the model. The first is the input-output architecture of materials from BEA's Use table. The Capital Flows table—also from the BEA—will inform the structure of capital purchases. Lastly, the network structure of intersectoral labor mobility will be assessed via the CPS microdata provided by the IPUMS project at the University of Minnesota. This latter data source will be novel to this paper, while the previous will closely follow the methodology of the aforementioned literature.

2.2.1 Sectoral Output

The BEA publishes yearly series on gross output by industry for up to 71 different industries at roughly three-digit NAICS disaggregation from 1997-2016. These series represent the total output in producer prices of a particular industry in a given year. (Note that this differs from GDP which would subtract value of intermediate inputs.)

Table 1 presents summary statistics of the growth rates of the output series. Weighted-average standard deviation of the growth rate is 2.7 percentage points while the average pairwise correlation between growth rates, $\bar{\rho}(y_i, y_j)$, is 0.143. The former value is slightly lower and the latter slightly higher than that reported in FSW, likely because they are using quarterly IP data while my series are yearly output.

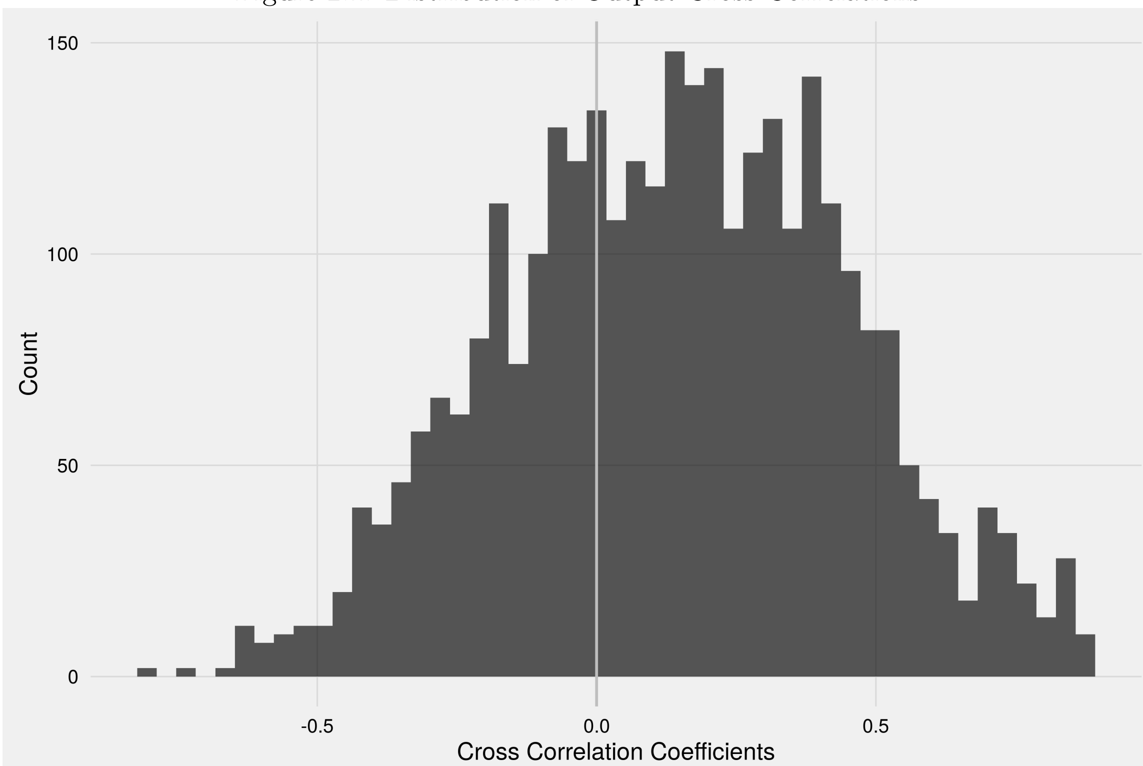
Table 2.1: Summary Statistics for Sectoral Output

Avg. Std Dev.	Avg. Cross-Correlation
2.7	0.143

The average correlation coefficient of the series is an important statistic for the analysis of output movement carried out in this paper. The estimation techniques to decompose volatility onto aggregate and sectoral sources will employ a structural, calibrated model to incorporate the effects of linkages within the economy. Examining whether the resultant model-generated cross-correlations are similar to the data will be important for determining whether it is accurately capturing the propagatory effects of linkages. In the results section this statistic will serve as a benchmark target to assess the fit of the different model calibrations.

The distribution of cross-correlation coefficients between output series is plotted in Figure 1. This distribution is visibly normal with a reasonably large standard deviation of 0.31. So while on average sectors are correlated, not all sectors are, nor is it a small number of highly correlated sectors that is driving up the average. The large standard deviation already suggests that sectoral shocks are active—if only aggregate shocks were important, one would likely think that output correlation coefficients would be both high and concentrated.

Figure 2.1: Distribution of Output Cross-Correlations



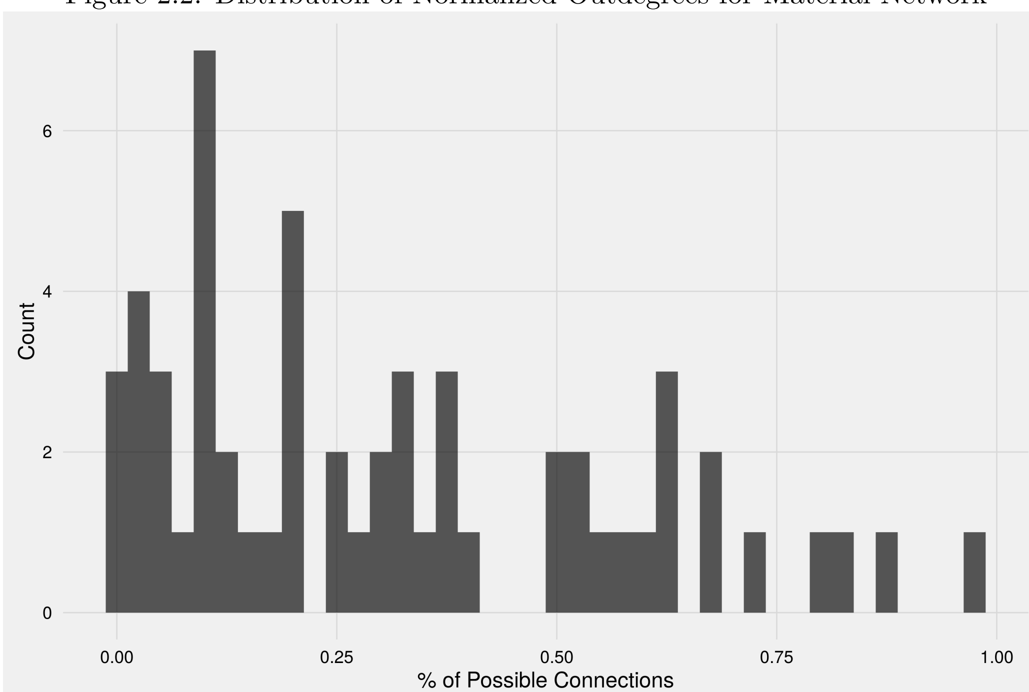
2.2.2 IO Architecture

The levels at which sectors purchase materials from each other is recorded in the BEA's Use table. A typical entry in this table gives the total amount spent in producer's prices by any given sector on another sector's output (as intput into their own production). The material use table induces a weighted, directed network $\Phi_M = (N, \varepsilon_M)$ where the N nodes are the economy sectors and the strength of the edges are the share of sector j's total purchases obtained from sector i.

As stated in the introduction, the material network represented by the Use Table will be key in calibrating the material requirements of firms within the model economy. The entries within the table give the long-run, steady-state average share of one sector's material goods in the production of another sector's and will therefore serve as the calibration target for this share in the model analog.

Figure 1 plots the histogram of the degree distribution of the material network. An edge is said to "exist" in this image if sector i provides more than 5% of sector j's out of sector purchases. The strong right-skew of the histogram portrays the presence of "star suppliers"—sectors which supply heavily to many others—which are shown in Acemoglu et al. (2012) theoretically and Foerster et al. (2011) empirically to be important in the amplification of sectoral shocks.

Figure 2.2: Distribution of Normalized Outdegrees for Material Network



2.2.3 Capital Flows

The Capital Flows table is the analog of the Use table for Capital goods and shows the purchases made by sectors on capital goods from other sectors. The right skewed nature of the degree distribution is even more pronounced in the capital flow table—most sectors provide 0 in capital goods, while Construction provides nearly 40% of the total.

2.2.4 Intersectoral Labor Movement

Material and capital goods s aren't the only things that flow between sectors. Workers as well reallocate between sectors as those sectors' ouput fluctuates. However, an analog of the IO table from the BEA does not exist for worker flows—that is, no table exists with the flows of workers from one sector to another.

This is potentially an important omission in any use of structural estimation that employs a multi-sector model. Take the example of a single sector propagating an experienced shock to the rest of the economy. The intuition is that if sector i experiences a positive shock, they will be able to increase their productive efficiency, hence making their goods cheaper. Since these goods are used as material inputs by other sectors, the shock will propagate in the form of cheaper materials to those sectors who buy from sector i. In this story, the extent to which sector i can capitalize on its positive shock is key to the quantity of gains it can realize and therefore pass on to its connected sectors.

However, a firm's ability to capitalize on a positive shock is affected by the kind of labor market it faces. When labor is purchased in an aggregate spot-market at an aggregate wage, individual sectors face a relatively flat labor-supply curve: unless the sector is very large, they can expect to be able to hire freely without much effect on the overall wage. However, if the sector faces frictions to increasing their labor supply—either generally through search frictions or because of a restriction in the pool of viable workers—the supply curve they face will be steeper and larger, idiosyncratic wage movements would be required. Therefore, because the possibility of labor frictions have implications for the propagation of shocks, it is important to asses whether frictions of this kind are present; and if so, incorporate them into the structural model used to filter the output data.

To accomplish the former, I consult the CPS microdata, obtained from the IPUMS database hosted at The University of Minnnesota. These data contain overlapping cohorts of approximately 50,000 individuals each of which questions are asked about the industry in which a respondent currently works. The structure of interviews involves two, segregated four-month panels: four months on; eight months off; then four months on again. I exploit this limited panel dimension to build an $N \times N$ matrix R of observed month-to-month intersectoral worker transitions over the same period of time as the output series, 1997-2016. Any entry in the matrix r_{ij}, shows the total number of transitions over the sample from sector i to sector j.

As identified in Hagedorn et al. (2017), the CPS microdata can often overstate industry switches due to noisy reporting. To combat this problem I employ the strategies suggested in Moscarini and Thomsson (2006)—namely, using a monthly rather than yearly timeframe, and using an algorithm to throw out likely spurious transitions. The following section details this filtering algorithm.

2.2.4.1 The Cleaning Algorithm

Moscarini and Thomsson (2006) constructed a filter that allowed them to discard likely spurious observations of occupational switches. I will consider a similar algorithm that discards spurious industry switches.

This begins by only considering data from 1994-2015. In 1994, the CPS began a new survey method for industry and occupation classification called "back-coding" that relies on asking if a worker is in the same job, rather than asking for independent job-characteristics every survey. This drastically reduces the number of observed (and likely erroneous) transitions over the pre-1994 to post-1994 samples. Additionally, I will be considering output data over the 1994-2015 period, so the 1994-2015 sample avoids any consistency issues related to changing patterns of labor mobility.

Another important issue to address with the CPS is that their sampling is based on fixed addresses, not households. IPUMS already matches households and individuals over time, meaning I do not have to worry about this issue. Though, if using raw CPS data I would have to account for changing household number—HHNUM.

With those preliminaries out of the way, the first level of the industry filter considers several variables that immediately signal a suspicious transition. These variables are AGE, SEX, and RACE. Clearly any of these changing over the sample (or in the case of AGE, changing by more than one year) indicates a highly suspicious sampling (the respondent has likely changed,) and so any industry switch thus observed must be discarded.

The second layer of the filter considers survey variables that are found to be highly correlated with occupation switches in Moscarini and Thomsson (2006), meaning that any transition without the accompanying correlates is considered suspicious. These variables are: EMPSAME—"Do you work for the same employer as last month?"; CLASSWKR—the class, e.g. public or private, of the worker's employing firm; and OCC1990—the occupation of the worker in the 1990 census classification. I employ an ANY3 method, whereby if none of the three correlated variables are of the correct value then the observed transition is thrown out. Precisely, if the data indicate an industry transition but 1) the occupation remains the same and 2) the worker is still working for the same firm and 3) the class of the worker remains the same I consider the transition spurious and remove it from the sample.

Lastly, I throw out the entire series for any individual who reports more than three total switches over the eight-month sample. This is a slightly different and perhaps somewhat stricter discriminant that Moscarini and Thomsson (2006) who only consider the first four-month sample for every individual, and therein allow certain samples with three total transitions but never four total.

After cleaning, I am left with 466,825 total industry switches. When standardizing all data sets to $N = 57$ sectors, this implies an average of 146.25 observations of switching per possible combination—representing good coverage. The average number of industry switches observed in a month—that is, the sum of off-diagonal elements over the total—is 3.4%, close to the number found in Moscarini and Thomsson (2006) for occupational switchers.

2.2.5 The Structure of Re-allocation

A natural question is whether the movement of workers between sectors is reasonably random, or whether it follows a definite pattern. A random realloaction of workers would imply that the probability of observing a worker move from sector i to sector j is given by the product of their sizes: $l_i l_j$ where l_i is the share of total workers employed in sector i. This is a simple multinomial model, and mimics the Balls and Bins model of Trade of Armenter and Koren (2014). To test the hypothesis of a multinomial model of random reallocation, for sector i let N_{ri} be the total number of workers transfering out of it in the sample. Then let $e_{ij} = N_{ri} \frac{l_j}{\sum_{k \neq i} l_k}$ be the expected number of transitions between i and j under the multinomial model. The following statistic can then be computed:

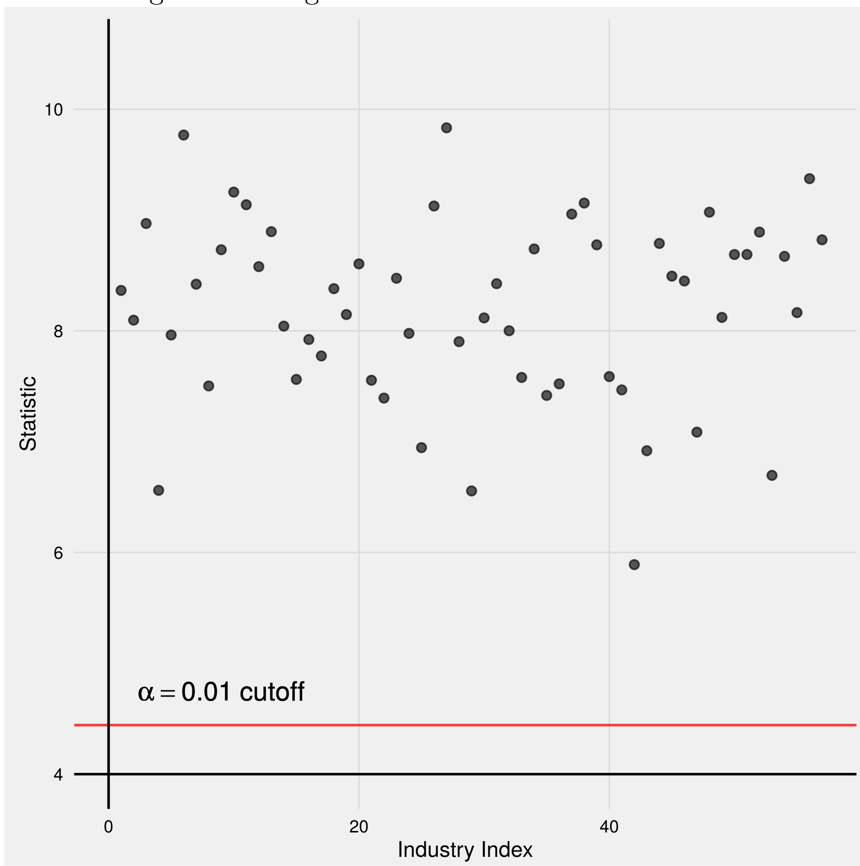
$$R_i = \sum_{j \neq i} (r_{ij} - e_{ij})^2 / e_{ij} \sim \chi^2_{N-1}$$

where r_{ij} is the observed number of transitions between i and j.

The logs of these statistics, which are distributed chi-squared with N-1 degrees of freedom, are plotted in Figure 2, along with the cutoff for rejection at the $\alpha = 0.01$ level. As is clear from the picture, for every sector in the sample, the null hypothesis of random re-allocation between sectors is rejected at the 1% level. The rejection also appears to be stronger for manufacturing sectors, with an average value of 65,370 vs. 30,691 for non-manufacturing sectors. This suggests that there is more explicit structure underlying the transition of employees between manufacturing sectors—an interesting topic for future work.

The results of the multinomial test show that sectoral re-allocation of labor contains a great deal of non-random structure. Because labor is an important input to production for virtually every sector, any model of a multisector economy—especially one that purports to filter out sectoral shocks from observable output data—should take account of this structure

Figure 2.3: Log Test Stat for Worker Reallocation



of re-allocation if it intends to have accurate properties.

The argument against previous models of sectoral shocks with flexible labor markets is similar to any model of flexible labor—firms must engage in time- and resource-consuming effort to match with workers and utilize their labor effectively. In the next section I endeavor to build a multisectoral model of the economy that can, in fact, account for this kind of frictional labor market and match its observables in the data. Along other dimensions, it will attempt to cater as closely as possible to previous literature so as to serve an accurate comparison.

2.3 Model

This section will construct a multi-sector model of an economy that can incorporate the features highlighted in the previous section. While the machinery used for the capital and material linkages will be similar to that seen in previous work, the movement of workers between sectors will be modeled according to a Lucas-Island style model where workers face probabilities of successfully transitioning between certain sectors. This will later be calibrated to match the observed transitions of workers from the CPS microdata.

2.3.1 Firms

Firms within the model will produce within an industry and each industry will operate as though with a representative firm. The output of a given industry can be used as material and capital inputs within their production according to a fixed technology that they own. Labor is recruited from within the island's labor pool at the prevailing island wage which is determined in equilibrium.

2.3.1.1 Final Goods

The economy is composed of N industries that can be interpreted as islands. Each industry j has a positive measure of firms who have access to a constant returns to scale, CES technology $f_j : \mathbb{R}^3_+ \to \mathbb{R}_+$ that maps labor, capital and material bundles into output according to:

$$f_j(\ell_{jt}, k_{jt}, M_{jt}) = y_{jt} = A_{jt}\left[\epsilon_j^{1-\psi_y}\ell_{jt}^{\psi_y} + \tau_j^{1-\psi_y}M_{jt}^{\psi_y} + \alpha_j^{1-\psi_y}k_{jt}^{\psi_y}\right]^{1/\psi_y}$$

where $\Omega_y = \frac{1}{1-\psi_y}$ governs the elasticity of substitution of final good production and the input weights are restricted such that $\epsilon_j^{1/\Omega_Y} + \tau_j^{1/\Omega_Y} + \alpha_j^{1/\Omega_Y} = 1\,\forall\, j$. A_{jt} is a factor-neutral technology which is modeled as a vector martingale process:

$$A_{t+1} - A_t = \varepsilon_{t+1} \quad \sim N(0, \Sigma_\varepsilon)$$

2.3.1.2 Inputs

The material bundle is created with material purchases from all other sectors according to a sector-specific technology:

$$M_{jt} = \sum_{i=1}^{N} \left[\gamma_{ij}^{1-\psi_m} m_{ijt}^{\psi_m}\right]^{1/\psi_m}$$

which is again CES in nature. m_{ijt} represents time t purchases of materials from sector i by sector j, while the matrix Γ with element γ_{ij} describes the importance of industries as producers for each other. The columns of $\Gamma^{1-\psi_m}$ are restricted to sum to 1.

I do not place any restriction or cost on the trade of islands with each other and allow material trade to occur simultaneously within a period. This contrasts with the setup of Long and Plosser (1983) who incorporate a one-period lag in material delivery (see FSW for a comparison between models).

Capital goods are a stock owned by firms which evolves according to $k_{jt+1} = (1 - \delta)k_{jt} + z_{jt}$ where

$$z_{jt} = \sum_{i=1}^{N} \left[\rho_{ij}^{1-\psi_k} x_{ijt}^{\psi_k}\right]^{1/\psi_k}$$

is investment in new capital. Like materials, investment is an aggregate of purchases from other sectors with the matrix P describing the input weights.

Note that for both capital and material goods, well-known aggregation results will imply that it does not matter who owns their production technology. I will interpret these input technologies as belonging to the final goods firms; though they could alternatively be thought of as separate firms operating in a Walrasian market with prices for their respective bundles.

Labor is recruited in a spot market from the stock of workers on an island. Let n_{jt} be the population of an island and L_{jt} labor per worker, then

$$\ell_{jt} = n_{jt} L_{jt}$$

The population on an island is fixed within a period. This implies that firms are only capable of increasing labor input on the intensive margin within the period. Any adjustment of labor on the extensive margin must come about through increasing the population of an island in future periods.

2.3.1.3 Prices

Market equilibrium will obtain under a list of prices taken as given by the agents: prices of each sector's final good, p_{jt}; wage in each sector w_{jt}; price of the material bundle in each sector p_{Mjt}; and the price of the consumption aggregate which will be assigned as the numeraire $P_t = 1$.

2.3.1.4 Decision Problem

The problem of a profit-maximizing final-goods firm in sector j amounts to:

$$\max \sum_{t=0}^{\infty} \beta^t \left[p_{jt} y_{jt} - w_{jt} \ell_{jt} - p_{Mjt} M_{jt} - \sum_{i=1}^{N} p_{it} x_{ijt} \right]$$

S.T.

$$y_{jt} = A_{jt} \left[\epsilon_j^{1-\psi_y} \ell_{jt}^{\psi_y} + \tau_j^{1-\psi_y} M_{jt}^{\psi_y} + \alpha_j^{1-\psi_y} k_{jt}^{\psi_y} \right]^{1/\psi_y}$$

$$M_{jt} = \sum_{i=1}^{N} \left[\gamma_{ij}^{1-\psi_m} m_{ijt}^{\psi_m} \right]^{1/\psi_m}$$

$$k_{jt+1} = (1-\delta) k_{jt} + \sum_{i=1}^{N} \left[\rho_{ij}^{1-\psi_k} x_{ijt}^{\psi_k} \right]^{1/\psi_k}$$

taking as given all spot prices. The aggregate state $S_t = \{s_{it}\}_1^N$ is made up of the industry state $s_{jt} = (n_{jt}, k_{jt}, A_{jt})$ of every island. This state is mapped into the next period according to the law of motion $S_{t+1} = \Phi(S_t)$.

2.3.2 Workers

In addition to a measure of firms, each island in a period has a population n_{jt} of workers who can sell their labor to firms in spot markets. These workers derive flow utility $U_{jt} = U(C_{jt})$ from consuming a CES aggregate of consumption goods that is bundled and sold by a positive measure of perfectly competitive firms:

$$C_t = \sum_{i=1}^{N} \left[\omega_i^{1-\psi_d} c_{it}^{\psi_d}\right]^{1/\psi_d}$$

where ψ_d is the elasticity of substitution between consumption goods in the consumption basket and ω determines the relative importance of the sectors' goods for consumption. Workers also experience disutility of labor according to $\frac{\Omega_L}{\Omega_L+1} L^{\frac{\Omega_L+1}{\Omega_L}}$

In each period, workers can attempt to switch sectors for the start of the next period. Each worker k receives a vector of idiosyncratic taste shocks $\mathbf{e}_{kt} = \{e_{k1t}, ..., e_{kNt}\}$ for the other sectors. They further face a probability π_{ij} of successfully making the transition between any two sectors i and j. The matrix Π contains these probabilities and is fixed. Workers then choose in which sector they will search for the period (including own).

The matrix Π is intended to help the model match the observable facets of the labor market observed in section two; namely, that the movement of workers between sectors is highly structured. I do not impose a strong interpretation on the values of π_{ij}, and will instead solely use them to attempt to match observable intersectoral flows between industry pairs. Their real-world referent could be either an observed probability of successfully finding a job in the new sector (perceived fit) or the probability of making contact with a firm from the new sector (informational). With the data I have it is impossible to identify between these two stories.

2.3.2.1 Decision Problem

The worker m in sector j observes the aggregate state, S_t as well as prices and its idiosyncratic profile of taste shocks, and chooses how much labor to sell, its volume of consumption bundles, and where to search during the period. Workers cannot save. Their bellman equation is then

$$\begin{aligned} V_{mj}(S_t) = \max_{\{C_{mjt}, L_{mjt}, k\}} & \{U(C_{mjt}, L_{mjt}) \\ & +\beta \mathbb{E} \left[V_{mj}(S_{t+1}) + \pi_{jk} \left(V_{mk}(S_{t+1}) - V_{mj}(S_{t+1}) \right) + e_{mkt} \right] \} \end{aligned}$$

$$\textbf{s.t. } C_{mjt} \leq w_{jt} L_{mjt}$$

This value function is standard apart from the search choice. The worker takes into account the probability π_{jk} of successfully transitioning into any of the other sectors from her current sector. So while a particular sector may have a particularly high expected value when on the island, V_{mkt+1}, it can be heavily discounted if the worker perceives a low probability of successfully transitioning. A worker in retail trade might love to get a job in the oil and gas extraction sector because of the concomitant high wages; however, they know that they are unlikely to succeed in this transition and will likely forego attempting it for more realistic options.

2.3.2.2 Aggregate Worker Flows

If one assumes that the taste shocks e_{mkt} are independently distributed according to a type I extreme value distribution (log-weibull) with variance ρ, and location parameter set to 0, then they can be integrated out to find a worker's transition probabilities between sectors. The probability of a worker in period t transitioning from sector i to sector $j \neq i$ ex ante taste shocks is given by

$$\theta_{ijt} = \pi_{ij} \frac{\exp\{\frac{\pi_{ij}}{\rho} D_{ijt}\}}{\sum_{k=1}^{N} \exp\{\frac{\pi_{ik}}{\rho} D_{ikt}\}}$$

where $D_{ijt} = \mathbb{E}\left[V_j(S_{t+1}) - V_i(S_{t+1})\right]$ is the expected gain of a successful transition. This is the well-known result from McFadden (1977) regarding discrete choice under Type I extreme value. The probability of transition between sectors for a given worker is an evaluation of the expected gains from such an attempt, relative to all other sectoral choices. This probability is made noisy by the parameter ρ, such that in the limit $\rho \to \infty$, the transition attempts are pure noise and the worker is equally likely to apply to any sector; and when $\rho \to 0$, a series of probability-augmented "no-arbitrage" conditions emerge to force the expected gains of a transition to 0. This limit is similar to the well-known Lucas island model with the transition probabilities being the only difference.

The parameter ρ can roughly be thought of as controlling how much sector-level wage/value differences direct the transitions in the labor market. However, it is important in another regard when it comes to fitting the data. Wage profiles within an industry are clearly not point masses, meaning that a given industry will have wage heterogeneity within it. ρ helps to capture this effect and could be thought of as representing variation in wage offers within the same sector. ρ also helps to reproduce the well known result in sectoral labor models that gross flows are much larger than net flows. This bespeaks a large amount of informational and wage variation in the labor market that is not captured by sectoral aggregates.

Once the taste shocks are integrated out in a worker's decision problem, an industry's population measure can now be modeled by a representative consumer with value function

$$V_j(S_t) = \max_{\{C_{jt}, L_{jt}\}} \left\{ U(C_{jt}, L_{jt}) + \mathbb{E}\left[\sum_i \theta_{jit} V_i(S_{t+1})\right] \right\}$$

where $\theta_{jjt} = 1 - \sum_{i \neq j} \theta_{jit}$. The model can now be solved without regard to idiosyncratic taste shocks.

The matrix of transition probabilities Θ_t describe a state-dependent first-order markov process for the labor flows between sectors. As such, the law of motion for island population can be written in vector form as:

$$n_{t+1} = \Theta_t' n_t$$

2.4 System Reduction and Approximation

The state space of the model becomes very large even for a small number of sectors. The minimal state space contains the capital stock k_{jt}, island population h_{jt}, and the productivity level A_{jt} for every sector in the economy. With $N = 57$ sectors this results in 171 state variables, all but requiring first-order perturbation methods.

Perturbation methods approximate the equilibrium behavior of a model by approximating its equations around a chosen point—typically the steady state of the model. First-order perturbation has some undesirable effects, most notably, it cannot capture curvature and therefore risk aversion of agents. However, this model is not meant to capture any realistic behavior related to risk aversion; indeed, there is no saving and households face risk only at the industry and aggregate level.

Even using perturbation methods, the model needs to be reduced to the minimal number of equations possible for reasonable computational speed. This section will perform such a "system reduction" to reduce the full set of model equations to a set of $4N$ dynamic equations in $4N$ unknowns—n_t, k_t, V_t, p_t.

2.4.1 Blanchard Kahn and Approximation

Equations x-y give a system of 4N equations in 4N unknowns in the familiar form for solving linear rational expectations models via the standard toolkits:

$$Ay_{t+1} = By_t + \Phi A_t$$

however, the model is under-identified at this point due to the normalization of the numeraire price $P_t = 1 \quad \forall t$. Therefore I use the system reduction algorithm outlined in King and Watson (1998) to account for static equations not readily apparent from the dynamic system. The first-order system is then solved using the standard King and Watson algorithm.

2.5 The Model Filter

Let $s_t = (n_t, k_t)$ be the minimal endogenous state used to execute the state-filtering algorithm. One can use the decision rules from the model approximation to filter the from the data the implied endogenous states and the exogenous series of exogenous shocks ε_t. To that end, the model solution gives us the following linear state space model with measurement and state evolution equation:

$$y_t = \Pi_a A_t + \Pi_s s_t$$

$$s_{t+1} = M_s s_t + M_a A_t$$

Π_a is invertible, allowing us to find an expression for the productivity series:

$$A_t = \Pi_a^{-1} \left[y_t + \Pi_s s_t \right]$$

Given the martingale assumption on A_t, we can then difference both of these equations to obtain

$$\varepsilon_t = \Pi_a^{-1} \left[\Delta y_t + \Pi_s \Delta s_t \right]$$

$$\Delta s_{t+1} = M_s \Delta s_t + M_a \varepsilon_t$$

From this system, one can then extract the model-implied disturbances from the observable output time series. Initializing $\Delta s_0 = \varepsilon_0 = 0$ to allows to iterate forward and filter the shocks ε_t for the duration of the output time series.

2.6 Factor Analysis

Once the implied sector disturbances are in hand, we can then estimate a common factor model to assess the relative contribution of sectoral vs. aggregate shocks. The productivity disturbances will by assumption be generated by the following common factor model:

$$\varepsilon_t = \Lambda F_t + u_t \qquad u_t \sim N(0, \sigma_u^2 I_N)$$

This common factor model posits the existence of sectoral vs. aggregate shocks. Every firm's productivity process is the combination of a common component—F_t with concomitant factor loadings λ_j for sector j—and an idiosyncratic component given by u_{jt}. As is typical in factor models, the covariance matrix of the idiosyncratic component is assumed to be diagonal.

Principal components can be used to consistently estimate the factors F_t, given the extracted series $\{\varepsilon_t\}$. While it is possible to use model-validation techniques to choose the number of common factors when estimating a common factor model, I will follow FSW and Atalay and estimate a two-factor model to ensure comparability to those papers. Mapping the extracted factor component $\Lambda \hat{F}_t$ and sector-specific component $\hat{u}_t$ back into the observable output series $\hat{y}_t$ over a large impulse response horizon gives an estimated covariance decomposition of $\hat{\Sigma}_{yy}$ onto aggregate and sectoral components respectively.

Now, recall the state space structure of the model:

$$\Delta y_t = \Pi_s \Delta s_t + \Pi_a \varepsilon_t$$

$$\Delta s_{t+1} = M_s \Delta s_t + M_a \varepsilon_t$$

$$\Rightarrow$$

$$\Delta y_t = \Pi_a \varepsilon_t + \Pi_s \sum_{i=0}^{\infty} M_s^i M_a \varepsilon_{t-i-1}$$

where the matrix power i means repeated matrix multiplication. A sufficiently large window of past shocks can then be used to assess the effect of shocks on output. Taking the variance of this expression then yields

$$\Sigma_{yy} = \Pi_a \Sigma_\varepsilon \Pi_a' + \Pi_s \left(\sum_{i=1}^{\infty} M_s^i M_a \Sigma_\varepsilon M_a' (M_s^i)' \right) \Pi_s'$$

After imposing the factor structure on the covariance matrix of the disturbances, one can now use the decomposition of Σ_ε with this impulse response representation to calculate $R^2(F)$:

$$\begin{aligned} \Sigma_{yy} =& \Pi_a \Lambda \Sigma_F \Lambda' \Pi_a' + \Pi_s \left(\sum_{i=1}^{\infty} M_s^i M_a \Lambda \Sigma_F \Lambda' M_a' (M_s^i)' \right) \Pi_s' \\ &+ \sigma_u^2 \left[\Pi_a \Pi_a' + \Pi_s \left(\sum_{i=1}^{\infty} M_s^i M_a M_a' (M_s^i)' \right) \Pi_s' \right] \end{aligned}$$

. This format gives the full decomposition of output variance into common and idiosyncratic components. The estimated factor and idiosyncratic model parameters can then be for the estimation procedure.

2.7 Calibration

The model's parameters need values assigned before its filter can be taken to the data. Several parameters are standard can be taken from the literature. β is set to 0.96 to correspond with the yearly frequency of the output time series. The depreciation rate δ is set to 0.1. $\bar{L}$ is normalized to 1. Ω_L—the elasticity of labor supply—is also normalized to 1 as σ will control the elasticity of labor, which is set to 0.7 to match a Frisch elasticity of 0.4.

The remainder of the parameters require some care in their calibration strategy. One goal of this section is to provide some historical comparability between the results in FSW, Atalay and this paper. Therefore, I will estimate four different calibrations of the model: one for each of the previous papers and two new strategies for the current one. The intent will be to progressively introduce new channels into the theoretical framework and assess their relative contribution as well as to bridge the gaps between former work. I will refer to the FSW and Atalay calibrations as M_{FSW} and M_{Atalay} respectively.

The calibrations of FSW and Atalay differ mainly in their targets for the production elasticities: Ω_y, Ω_d, Ω_m and Ω_k. FSW somewhat agnostically uses unitary elasticities at all levels of production, in the absence of any data on the responses of firms to prices. Atalay, by contrast, attempts to directly estimate these elasticities using an instrumental variables approach. That paper finds vastly lower elasticities of substitution than unitary especially for the material elasticity, Ω_m which is estimated at close to 0. For Atalay's calibration I will use $\Omega_y = 0.5$, $\Omega_m = 0.1$, and $\Omega_d = \Omega_k = 1$.

Several share parameters in the model need to be calibrated. Γ and P will be calibrated to match the expenditure share of sectors upon each others' goods in both the material bundle and the investment bundles respectively from the Use table and Capital Flows table. α, τ and ϵ will be calibrated to match the income/expenditure share of each sector on investment, material purchases, and wage payments respectively relative to total input purchases—this information is also available in data from the BEA.

Another key difference is present in the calibrations of FSW and Atalay, regarding the importance of capital in production: α. The chosen calibration target for the expenditure share of capital in FSW is total value added, meaning that it includes profit (or returns to investors), while the latter paper takes only purchase value of new capital expenditures as the data analog. Profits to firms should be considered part of income accruing to capital; however, including this when calibrating is somewhat contrary to the nature of α in the model. α represents the importance of the *concrete* capital goods purchased from other sectors. Increasing α by including returns to investors overstates the importance of these production inputs in the production function. Because neither interpretation is obviously correct, and to improve comparability with both papers, I include calibrations with both statistics.

The probabilities of successful movement between sectors, stored in the matrix Π will be calibrated to match the observed movement between sectors in steady state. Because the observable output series are at a yearly frequency, the Markov transition matrix of observed transition proportions from the data (which are monthly) must be multiplied out for twelve periods.

In order to assess the relative contribution of the general friction to switching sectors (workers cannot reallocate freely within a period and must wait one period no matter their target sector) and the network frictions where workers face probabilities of successful transition, I will include two calibrations. In the first, the network frictions to switching sectors are shut down, that is, $\pi_{ij} = 1$ for all sectors. Call this calibration M_{open}. The second calibration is the full model with the π_{ij} calibrated to match observed transitions. Call this model M_{ind}

2.8 Results

In order to contextualize the results of the model, I will compare the relevant statistics to calibrations of the model that conform to past exercises in the literature—namely, FSW and Atalay. Both of these models involve shutting down frictional labor—meaning that labor is hired in an aggregate spot market at a single wage each period. (See either of these papers for details). The two models differ from each other in their production elastiticities, as detailed in the Calibration section.

It should also be noted that Atalay presents a different set of statistics than those presented here and in FSW. In that paper and this one, sectoral shocks are mapped, via the model filter, back to observables, and statistics are reported for those values. Atalay reports the statistics only for the underlying shocks and not the residuals. Here I have reported the statistics for the Atalay calibration as those for the observables, which might explain some discrepancies.

Table 7 presents several statistics for the model-based output growth rates under several calibrations and the FSW and Atalay benchmarks. The first statistic—$R^2(F)$ is the proportion of aggregate, observable output volatility that is attributable to common factors. This is the statistic reported in FSW, and I follow them in allowing for two underlying common shocks. Note that Atalay performs his analysis using one underlying factor. This naturally adjusts $R^2(F)$ downward as fewer underlying factors are allowed to account for the variation in the time series. However, the results seem to be relatively robust to upward adjustments in the number of factors above two (as found in FSW).

The first comparison to note is that of M_{FSW} vs. M_{Atalay}. The difference in these two models is the reduction in the elasticity of inputs for final goods and material bundles from 1 to 0.5 and 0.1 respectively. The statistics for both models accord broadly with those found in their papers. Note, however, that the choice of calibrating data statistic for capital shares is important for all three reported statistics.

The results for contribution of aggregate shocks in both these models are similar to those reported in Atalay. For that model, the statistics for both correlation of sectoral output and volatility of aggregate output are both much larger than found in the data. Apparently, the model is generating output that is too correlated relative to data, and therefore overall volatility is too high (overall variance being the weighted sum of all variances and covariances). This presents a puzzle—given the estimation of Ω_y and Ω_m in Atalay—one that will be partially resolved by incorporating realistic labor frictions.

Comparing the M_{atalay} model with the M_{open} model, we see that incorporating labor as a state variable with island frictions revises upward the estimate of the contribution of sectoral shocks. The M_{Atalay} calibration estimates a contribution of common shocks of 0.04 and 0.21 for the two capital calibrations. For the M_{open} model, this statistic is revised upward to 0.32 and 0.46. This change reflects the inability of firms to adjust their labor supply as flexibly in a Lucas-type model.

The average correlation of output growth is lowered considerably by incorporating island frictions. This is due to the differences in the way labor is recruited between the models. When labor is perfectly flexible, there is a single wage determined throughout the whole economy, and firms are free to hire as much or as little as they prefer at that wage. In the island model, firms are stuck with their island population in the short term. This means that a sector that would like to increase production in good times is hobbled by a rapidly increasing wage as any increase in labor at the sectoral level must occur on the intensive margin. Obversely, when times are bad, a larger than needed population of workers remain on the island, implying that labor is cheaper than it once was, offsetting the decreased productivity. Overall this effect serves to hobble the propagation of sectoral shocks through the material network, especially in the case of low elasticities.

Moving right on the table, the next comparison is between the open island model and the M_ind model, where worker transition probabilities between sectors are calibrated to match observed transitions. Surprisingly, incorporating realistic transition probabilites does little to change the chosen statistics to the model—all three remain virtually unchanged. This suggests that the labor market is relatively robust in apportioning labor, despite the observed sparsity in pairwise flows. Several papers have suggested a "mismatch" theory of unemployment; it appears that, at least as it applies to sector-specific skills and re-allocation, mismatch is not a large determinant of labor market inefficiency.

Perhaps the best way to couch this latter result and to provide a window into further

Table 2.2: Model Comparison

Calibration	$R^2(F)$	$\bar{\rho}$	$\bar{\sigma}_g$
	$\mathbf{M_{FSW}}$		
Cap Expend.	0.49	0.17	3.51
Value Add	0.77	0.22	3.57
	$\mathbf{M_{Atalay}}$		
Cap Expend.	0.04	0.59	10.18
Value Add	0.21	0.45	6.02
	$\mathbf{M_{Open}}$		
Cap Expend.	0.32	0.37	4.58
Value Add	0.46	0.31	3.89
	$\mathbf{M_{Ind}}$		
Cap Expend.	0.32	0.38	4.62
Value Add	0.46	0.31	3.90
	Data		
-	-	0.143	2.7

work is to consider the model as solved by the social planner. Given the lack of externalities in the model, the social planner's solution will match the competitive equilibrium. It is not hard to imagine that, even on a very sparse network of labor reallocation, a social planner would be adept at correctly apportioning labor reallocation along network links. It remains interesting that the network is robust despite its sparsity however. It would be interesting to consider an environment without such strong coordinating features, however; perhaps with a lag in information updating by firms and/or workers.

2.9 Conclusion

This paper incorporated realistic stucture for all three production inputs in a multisectoral real business cycle model in order to estimate the contribution of aggregate vs. sectoral shocks. In so doing, it documented the structure of sectoral labor reallocation and calibrated a structural model to match the features of that structure. The structural model generated a filter that was used to estimate the contribution of the two different sources of disturbances from sectoral output data in the US economy.

Approximately half of aggregate volatility is due to sectoral disturbances—a higher number than in past literature. The mechanism responsible for the revised estimate vis a vis past literature is a general sluggishness in the adjustment of labor and not the particular structure of sectoral labor reallocation—which is largely irrelevant when general frictions are incorporated.

Future research could advance in several directions. Realistic unemployment is absent from the model, and workers would make different re-allocation decisions based on whether they were unemployed or not. Second, the model above imposes the same elasticities in production for all industries. This is partially due to paucity of data; but there might be some value to attempting to estimate different elasticities for different industries, given how much these elasticities can change the main results.

Appendix A: Model Approximation

The key first order conditions from the firms' and consumers' decision problems are:

$$w_{jt} = p_{jt}\left((1-\tau_j)\frac{y_{jt}}{\ell_{jt}}\right)^{\frac{1}{\Omega_y}}$$

$$P_{Mjt} = p_{jt}\left(\tau_j\frac{y_{jt}}{M_{jt}}\right)^{\frac{1}{\Omega_y}}$$

$$p_{it} = P_{Mjt}\left(\gamma_{ij}\frac{M_{jt}}{m_{ijt}}\right)^{\frac{1}{\Omega_m}}$$

$$c_{jt} = p_{jy}^{-\Omega_d}\omega_j C_t$$

$$\frac{1}{C_{jt}} = \frac{L_{jt}^{1/\Omega_L}}{w_{jt}}$$

$$\mu_{jt} = p_{jt+1}\left[\alpha_j\frac{y_{jt+1}}{k_{jt+1}}\right]^{\frac{1}{\Omega_Y}} + \beta(1-\delta)\mu_{jt+1}$$

Firm Equations

I will begin the log-linearization and system reduction at the firm's supply equations. In the following sections a tilde above a variable denotes percent deviation from steady state.

Variables without industry subscripts e.g. y_t are understood to be the vector of industry variables $(y_{1t}, ..., y_{Nt})$

First, let $X_l = 1 + \Omega_Y \left(\sigma + \frac{1}{\Omega_L}\right)/(1-\sigma)$ represent the wage-labor tradeoff. And ξ_l, ξ_k, and ξ_n represent the steady state marginal products of the material bundle, capital and labor respectively. Substituting the optimal policy for the material bundle into the supply equation yields:

$$\Pi_{yy}\tilde{y}_t = \Pi_{ya}\tilde{A}_t + \Pi_{yp}\tilde{p}_t + \Pi_{yk}\tilde{k}_t + \Pi_{yn}\tilde{n}_t$$

where

$$\begin{aligned}
\Pi_{ya} =& \text{diag}((1-\tau\xi_l^{1-\Omega_y})^{-1}) \\
\Pi_{yp} =& \text{diag}((1-\tau\xi_l^{1-\Omega_y})^{-1}) \\
\Pi_{yk} =& \text{diag}((1-\tau\xi_l^{1-\Omega_y})^{-1}\alpha\xi_k^{1-\Omega_y})) \\
\Pi_{yn} =& \text{diag}((1-\tau\xi_l^{1-\Omega_y})^{-1}(X_l-1)\epsilon\xi_n^{1-\Omega_y}/X_l))
\end{aligned}$$

Then, the supply equation of firms can be written as

$$S_{yy}\tilde{y}_t = S_{yp}\tilde{p}_t + S_{yk}\tilde{k}_t + S_{yk1}\tilde{k}_{t+1}$$

Substituting for $\tilde{y}_t$ from the above equation yields the capital accumulation equation.

Euler Equations

Letting $\Delta_{pz} = \text{diag}(p_{Mss}^{\Omega_m - 1})\left[\Gamma\text{diag}(p_{ss}^{1-\Omega_M})\right]'$, and substituting for $\tilde{y}_{t+1}$ from the supply equation, we can get

$$\Gamma_p\tilde{p}_t = \Gamma_a\tilde{A}_{t+1} + \Gamma_k\tilde{k}_{t+1} + \Gamma_n\tilde{n}_{t+1} + \Gamma_{p1}\tilde{p}_{t+1}$$

Labor Dynamics

From the state-dependent law of motion of island population equations, we can get

$$\tilde{\theta}_t = T_V \tilde{V}_t + T_n \tilde{n}_t$$

where $\tilde{\theta}_t = (\tilde{\theta}_{11t}, \tilde{\theta}_{12t}, ..., \tilde{\theta}_{1Nt}, \tilde{\theta}_{21t}, ...\tilde{\theta}_{NNt})t$ with appropriate Log-linearized coefficients.

Chapter 3

Product Search, Markups and Variety

3.1 Introduction

At the turn of the 20th century, coal mining towns throughout the United States would often feature a company store. In the folk imagination, these stores were notorious for poor selection, high prices and indebted patrons. However, once automobile use became mainstream and coal miners could go to town to shop, the company store quickly disappeared. The company store is a symbol for how search-constrained shoppers allow the existence of high prices and poor variety-selection. Its disappearance suggests that, as new search technology becomes available to consumers, the pricing and establishment allocations of an economy are forced to change as well.

The ability of consumers to view prices is currently undergoing a new transition. Figure 3.1 shows the percentage of all retail sales that occur online in terms of dollars sold. Starting under 1% in 2000 it is now set to surpass 9% in 2017, with 80% of all US adults now shopping online in some form or another. Evidently, shoppers are seeing an increasing ability to quickly and efficiently compare goods on price and quality and make the purchase

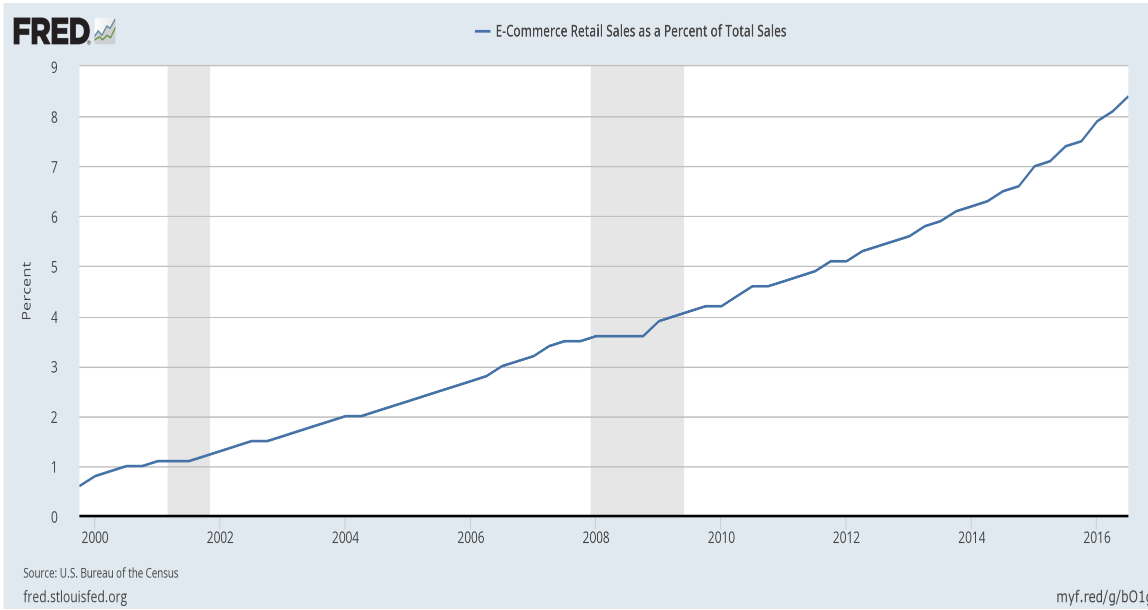


Figure 3.1: Ecommerce

that is best for them.

One of the most thought-provoking questions to emerge in the face of online retail and the search technology it offers consumers is "How will prices be affected?" Straightforward intuition suggests that as consumers are better able to sample prices and choose the lowest cost good, prices should go down in response to online search. However, Ellison and Ellison (2018) show that online prices are actually higher in the market for used books—the more so the more special interest the title.

This paper shows that both outcomes—rising or falling prices–can be consistent with online search within a competitive general equilibrium. The key feature of products that determines the response of their prices to search is how consumers' tastes are distributed. If a product is relatively generic, meaning that consumers' valuations are clustered tightly around the mean, then prices will decline with search as firms compete more strongly. However, if the taste distribution is highly right-tailed—corresponding to very specialized interests over the goods—then increasing search will cause firms to focus their pricing strategy on a small subset of consumers that value them highly. This is exactly the outcome observed in Ellison and Ellison (2018)—where a highly specialized product goes from being sold to a large mass of relatively uncaring consumers to one where it is priced for a small number of consumers who value it highly. This paper provides an exact characterization in terms of the taste distribution of when prices will increase or decrease with respect to search.

Concurrent with the persistent increase in online sales has been an emergent concentration of production in smaller numbers of firms. Figure 3.1 displays the number of employees per firm from data gathered by the Statistics of US Businesses dataset provided by the BEA. Firms are evidently facing strong impulses to concentrate production, and I document in this paper that an increase in consumer search behavior can explain this result. This effect will come about predominantly through downward pressure on prices, and is likely to affect industries with more generic products strongly. Indeed, according to the model, more specialized products like used books should see an increase in the number of producers as the same forces that increase prices will induce new entrants to the market. While used books by definition preclude this kind of market entry, one can take as an example the massive proliferation in YouTube™channels as a demonstration of this effect. When consumers are free to search at low costs, one should expect increases of varieties in highly taste-sensitive areas.

To demonstrate the relationship between search and prices and market entry/exit, I will first build a simple and intuitive shopping space where firms and consumers interact. This space will have the flavor of Burdett and Judd (1983) and more recently Kaplan and Menzio (2016): where differential information for consumers motivates the pricing decisions of firms. Unlike those papers, I will endogenize the search decisions of consumers and make them flexible, such that consumers can choose the expected size of the basket of prices they view. I will demonstrate that increased search generates two effects: a competition effect and a match-quality effect. The former serves to depress prices while the latter pushes them up; and the dominant effect (based on the shape of the distribution from which consumers

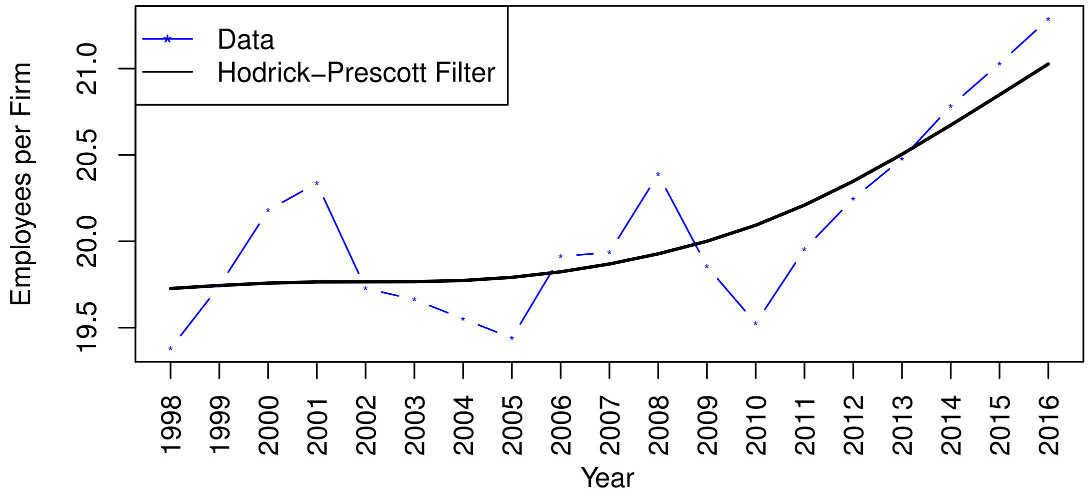


Figure 3.2: Employees Per Firm

draw their taste) determines the overall response of price levels to cheaper search.

The tractability of the shopping space will then allow me to embed it into a general equilibrium model that features entry and exit by firms. I will demonstrate the comparative effects of online search within the context of this model, showing how the response of markups, sales and firm entry/exit respond to increased search based on the tastes within an industry. While right-tailed products will see a large proliferation of varieties in response to cheaper search, more mean-centered products can see either declining or non-monotonic responses.

The rest of the paper proceeds as follows. Section 2 will present the shopping space in which consumers and firms interact. It will present the theoretical results on pricing responses to search. Section 3 will present general equilibrium results in light of the shopping architecture and Section 4 concludes.

3.2 The Model

3.2.1 The Shopping Space

Consumers and firms within the model interact according to an explicit microstructure subject to search frictions. The environment should be considered as a single industry, where the goods are largely substitutable up to an idiosyncratic, match-specific taste draw that is drawn by the consumer for each variety produced by firms. Every firm produces a distinct variety, meaning that every consumer has a vector of tastes representing their unique valuations over the different goods.

In order to ascertain the location of these distinct varieties, consumers have access to a search technology $\phi(;z)$, parameterized by a search efficiency parameter z. As stated earlier, $\phi(;z)$ is a pmf over number of goods sampled from a population of firms, each selling a unique variety. A well-known example of this kind of search technology is the Burdett Judd (1985) environment, where $1-\phi(1;z) = z = \phi(2;z)$. Another example that I will favor throughout this section is a poisson distribution where z represents expected number of draws

$$\phi(k;z) = \frac{e^{-z}z^k}{k!}$$

. z is a choice variable by the shopper, such that they can pay a cost to increase the number of varieties they are likely to view.

Consumers' tastes over the varieties offered by the firms are drawn from a taste distribution described by the CDF $G()$, and they are hidden from the consumer until a specific variety is observed. A consequence of learning valuations on observations is that the level of a consumer's taste shocks will not affect their incentives to search. Other than the taste draws, products within an industry are homogeneous and so consumers will only want to consume one good out of their industry basket.

3.2.2 Sellers

Imagine a producer facing the problem of selling to consumers within an industry. The producer takes as given the demand function of the consumer $D(\tau, p, \mu)$ and the indirect utility function $V(\tau, p, \mu)$ over the taste, price, and shadow value of the numeraire for a consumer. Define the function $h(\tau, p, \tilde{p})$ to be the function such that, if a consumer views two products in an industry with prices $p, \quad \tilde{p}$, $V(h(\tau, p, \tilde{p}), p, \mu) = V(\tau, p, \mu)$. That is, $h(\tau, p, \tilde{p})$ is the taste value that would make the consumer indifferent between the two products.

I will detail the producer's problem, working backward in time, to derive the objective function. First, let the taste profile of a consumer have an arbitrary ranking $\{\tau_0, ..., \tau_{N-1}\}$ over the N varieties in the industry where τ_0 is the highest. The probablity that a shopper who has viewed k prices would prefer the ith ranked object out of their basket (i.e. i was their highest draw) is

$$\left(\left(\frac{N-i}{N}\right)^k - \left(\frac{N-i-1}{N}\right)^{k-1}\right)$$

. This is essentially $Pr(j \leq i) - Pr(j \leq i-1)$ from an ordered multinomial draw.

The probability of a variety being the ith ranked for a consumer, given that the taste shock for that variety is τ, is then a binomial over the $N-1$ other varieties given by

$$\binom{N-1}{i} G(\tau)^{N-1-i}(1-G(\tau))^i$$

Then, with the distribution over prices viewed given by $\phi(k; z)$, the probability of a sale (and the expected number of sales) of a producer with price p and taste τ, given all other firms charging $\tilde{p}$ is given by

$$\sum_{k=0}^{\infty} \phi(k; z) \sum_{i=0}^{N-1} \binom{N-1}{i} G(h(\tau, p, \tilde{p}))^{N-1-i}(1-G(h(\tau, p, \tilde{p})))^i \left(\left(\frac{N-i}{N}\right)^k - \left(\frac{N-i-1}{N}\right)^{k-1}\right)$$

When $\phi(k;z)$ is a poisson pmf, the above reduces to

$$\tilde{s}(\tau,p,\tilde{p}) \equiv \left(1-e^{-z/N}\right)\left[(1-e^{-z/N})G(h(\tau,p,\tilde{p}))+e^{-z/N}\right]^{N-1}$$

It is worth taking a moment to unpack the above expression. The first term merely represents the probability of being viewed by consumers, and so the effect of more search, in the form of z increasing, is to improve the sales of the firm as more shoppers overall become aware of them. The second term displays the competitive effects that a firm will face when trying to sell over other firms. When z is low, that is the consumer is in expectation not viewing very many prices, the sale probability (conditional on having been viewed) will be heavily weighted toward one. This is the case where a firm is reasonably confident that any shopper is not viewing or actively considering any other firms' prices, and so the weighting on the ranking of τ, given by $G(h(\tau,p,\tilde{p}))$ does not enter strongly in the probability.

In the obverse, as z increases and the consumer is viewing more and more prices, the competitive effects begin to be weighted much more strongly and the seller has to beat out many other competitors, hence a strong weight on $G(h(\tau,p,\tilde{p}))$. In the limit, as $z\to\infty$, the probability of sale approaches

$$\tilde{s}(\tau,p,\tilde{p}) = G(h(\tau,p,\tilde{p}))^{N-1}$$

which is the perfect information, pure product differentiation model of Perloff and Salop (1985).

Note in addition that the internal weighting of the above probability depends primarily on the ratio $\frac{z}{N}$, or the search-variety ratio. So, the transition from a noisy to a perfectly informed world will require more search when there are more total varieties. Taking the number of varieties to a continuum in the limit, one finds

$$\lim_{N\to\infty} \tilde{s}(\tau, p, \tilde{p}) = e^{-z(1-G(h(\tau,p,\tilde{p}))}$$

.

The function $\tilde{s}$ describes the probability of selling to a consumer of a given taste variety τ, the next section will use this to determine the firm's decision problem and derive a symmetric pricing rule thereof.

3.2.3 The Firm's Objective

This section will detail the firm's program vis-a-vis their price setting behavior. It will derive the equilibrium condition for a symmetric pricing rule, and will demonstrate the two main theorems of the paper for the effect of search on prices. The crux of these two theorems is that the shape of the taste distribution for consumers is what determines the effect of search on prices.

A firm will engage in an N player game with the other variety sellers in order to maximize its profits, given the state of the world. The firm owns a constant returns to scale production function for its variable production, implying a marginal cost c. Also assume that all consumers share a shadow value of a numeraire μ The firm's problem is to maximize its profits:

$$\max_{p} \quad (p-c) \int_{supp(G)} D(\tau, p, \mu) \tilde{s}(\tau, p, \tilde{p}) dG(\tau) \tag{FP}$$

where $supp(G)$ represents the support of the taste distribution.

Taking first order conditions, and assuming a symmetric equilibrium, (and noting that $h(\tau, p, p) = \tau$) yields the following (implicit) pricing strategy for a firm. (There is a mild assumption here about differentiability of demand that has to be addressed when demand is non differentiable.)

$$p - c = -\frac{\int_{supp(G)} D(\tau, p, \mu)s(\tau)dG(\tau)}{\int_{supp(G)} D_2(\tau, p, \mu)s(\tau)dG(\tau) + \int_{supp(G)} D(\tau, p, \mu)s_p(\tau)h_2(\tau, p, p)dG(\tau)}$$

where $s(\tau) \equiv \tilde{s}(\tau, p, p)$ and $s_p(\tau) = \frac{\partial}{\partial p}\tilde{s}(\tau, p, \tilde{p} = p)$

There two relevant incentives for the expressed in the denominator above. The first is standard, that is, how much marginal loss in demand a firms loses from a marginal price change, given that a consumer decides to buy from that firm. The second represents the marginal decrease in *probability* of making a sale from a marginal price change in expectation.

Markups are then determined by the marginal effects of price changes on demand conditional on sale and on probability of sale. The effect of increasing search, however, will come about through the effects on $s(\tau)$, or the probability of sale. For the remainder of this section I will detail two theorems for how prices will affect search under two specific parameterizations of utility, linear and multiplicative in taste and quantity.

3.2.4 The Linear Case

Suppose that the utility of the consumer is linear when one unit is consumed, and for now as well assume that the shadow value of a dollar is equal to one. That is,

$$V(x, \tau, y) = \tau\{x = 1\} + y(M - p)$$

$$\Rightarrow D(\tau, p, 1) = \begin{cases} 1 \iff \tau \geq p \\ \\ 0 \iff \tau < p \end{cases}$$

. One can then rewrite the producer's problem as:

$$\max_p (p - c) \int_p^\infty \tilde{s}(\tau, p, \tilde{p})dG(\tau)$$

Which will deliver the following condition (once symmetry is imposed) that pins down the pricing rule in any equilibrium:

$$\begin{aligned}
&\int_p^\infty s(\tau)dG(\tau) + (p-c)\left[-s(p)g(p) + \int_p^\infty s_p(\tau)dG(\tau)\right] = 0 \\
\Longleftrightarrow &\int_p^\infty s(\tau)dG(\tau) + (p-c)\left[-\int_p^\infty g'(\tau)s(\tau)d\tau\right] = 0 \\
\Longleftrightarrow\, & p = c + \frac{\int_p^\infty s(\tau)dG(\tau)}{\int_p^\infty g'(\tau)s(\tau)d\tau}
\end{aligned}$$

where integration by parts was used on the term in brackets for the second line.

At this point it is useful to define the following function:

$$\tilde{G}(z) = \left[(1 - e^{-\frac{z}{N}})G(\tau) + e^{-\frac{z}{N}}\right]^N$$

which generates the CDF

$$\hat{G}(\tau) = \frac{\tilde{G}(\tau) - e^{-z}}{1 - e^{-z}}$$

Lemma 3.2.1. *$\hat{G}(\cdot; z)$ first-order stochastically dominates $\hat{G}(\cdot; z')$ if $z > z'$*

Proof. $e^{-\frac{z}{N}}$ is decreasing in z. □

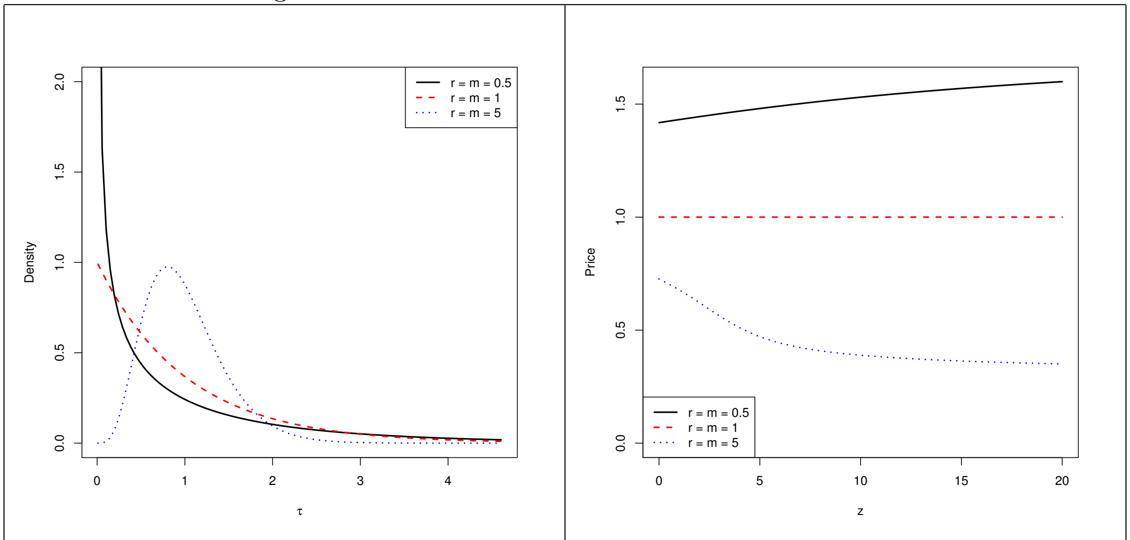
This lemma will be instrumental in proving the following proposition that shows the precise response of prices to search:

Proposition 3.2.2. *When utility is linear in quantity and taste, the consumer purchases one unit, and tastes are $\sim \Gamma$ (d,λ) then the symmetric price equilibrium is increasing (decreasing) in z if and only if m is greater (less) than 1*

Proof.

$$g'(\tau) = -rg(\tau) - \frac{1-m}{\tau}g(\tau)$$

Figure 3.3: Gamma Densities and Price Schedules



Γ Densities

Response of Prices to Search

Rewriting the pricing rule:

$$
\begin{aligned}
p &= c + \frac{\int_p^\infty s(\tau)dG(\tau)}{r\int_p^\infty s(\tau)dG(\tau) + (1-m)\int_p^\infty \tau^{-1}s(\tau)dG(\tau)} \\
&= c + \frac{1}{r + \frac{(1-m)\int_p^\infty \tau^{-1}s(\tau)dG(\tau)}{\int_p^\infty s(\tau)dG(\tau)}} \\
&= c + \frac{1}{r + (1-m)\mathbb{E}_{\hat{G}}[\tau^{-1}|\tau \geq p]}
\end{aligned}
$$

Using lemma 3.2.1, we see that $\mathbb{E}_{\hat{G}}[\tau^{-1}|\tau \geq p]$ is decreasing in z, which shows p increasing in $z \iff m > 1$, as desired. $\square$

Figure 3.3 shows example gamma densities for different parameterizations, as well as the response of prices to an increase in search by consumers. As proposition 3.2.2 states, prices increase with search when $m < 1$ and decrease otherwise. The left figure shows how m, the shape parameter of the gamma distribution, affects the overall shape. Low values of m are very right-tailed, with the exponential distribution corresponding to $m = 1$ as the case with constant prices. Overall, the exponential distribution is still very right-tailed relative to other parameterizations or other distributions—meaning that a heavily right-tailed taste distribution is required to actually observe increasing prices with search. For example, the next corollary proposition shows that prices cannot increase in the linear utility environment when tastes are modeled as a Gaussian distribution.

Proposition 3.2.3. *When tastes are distribution $\mathcal{N}(0,\sigma^2)$ in the linear utility case, prices can only decrease with search*

Proof.

$$g'(\tau) = -\frac{\tau}{\sigma^2} g(x)$$
$$\Rightarrow p = c + \frac{\sigma^2}{\mathbb{E}_{\hat{G}}[\tau | \tau \geq p]}$$

□

This result is perhaps unsurprising given that the normal distribution is the limit of a gamma distribution as the shape $m- > \infty$.

3.2.4.1 Competition vs. Concentration

While the proof establishes the desired result, it does little to demonstrate the intuition at play. For this, it is useful to return to the firm's problem and recast it in a more general form. Again consider a firm hoping to sell to a given consumer with valuation τ, rewrite $\tilde{s}(\tau, p, \tilde{p})$ as

$$\tilde{s}(\tau, p, \tilde{p}) = \pi(\pi G(\tau - p + \tilde{p}) + (1 - \pi))^{N-1}$$

this is the generalized form found when the probability of viewing any given price is π. In the specific example above with a Poisson distribution over prices viewed, we have $\pi = 1 - e^{-\frac{z}{N}}$, or 1 minus the probability of not viewing the price with a rate of $\frac{z}{N}$. This expression has the following explanation from the perspective of a seller: the first π is the probability of one's own variety being viewed, then, the probability under the exponent is the probability of beating out any other variety. This is done with certainty if the other variety was not viewed at all—an event with a probability of π—and with probability $G(\tau)$ in the event the other price was viewed. The producer must beat out $N-1$ such other varieties, and so this probability is raised to that power.

At this point one can differentiate this probability with respect to p and set $\tilde{p} = p$ to discern the marginal effect of price increases on a firm's probability of sale in a symmetric equilibrium:

$$s_p(\tau) = -\pi(N-1)(\pi G(\tau) + (1-\pi))^{N-2} g(\tau)$$

. Increasing the price of one's good induces the possibility that any of the $N-1$ other varieties would supplant one's own variety in preference, which is the case for the mass given by those who beat out $N-2$ other varieties times the mass $g(\tau)$ who are indifferent between one's own and another variety–this mass can be considered those consumers for whom the firm is actively contending with another firm.

One can now rewrite the first order condition from the firm's problem in the more generalized setup as:

$$\int_p^\infty s(\tau) dG(\tau) + (p-c)\left[-s(p)g(p) - \pi(N-1)\int_p^\infty (\pi G(\tau) + (1-\pi))^{N-2} g(\tau) dG(\tau)\right] = 0$$

. The firm considers the negative effects on demand from increasing prices on each of the possible valuations of its good, weighted by the distribution function G. I will now turn to the effects of search on the function $\tilde{s}$ to demonstrate how probabilities of sale change when search increases. Then I will demonstrate through the cross partial of prices and search how a producer's incentives over price changes themselves change as search increases.

In this environment, the probability π is isomorphic to the number of prices viewed by consumers, so we can observe the effect of increased search on $\tilde{s}$ by differentiating with respect to π:

$$s_\pi(\tau) = (\pi G(\tau) + (1-\pi))^{N-1} - (N-1)(\pi G(\tau) + (1-\pi))^{N-2}(1-G)\pi$$

. There are two effects of an increase in search on the probability of sale. The first term is the increased probability that a consumer who prefers the firm in its basket will actually find the firm. The second term represents the marginal probability that a consumer will discover an erstwhile unknown variety that they prefer to the firm's.

Now, differentiating again to find the cross partial of prices and search on the probability of sale delivers the following expression:

$$
\begin{aligned}
s_{p\pi} &= -(N-1)(\pi G(\tau) + (1-\pi))^{N-2} g(\tau) + \pi(N-1)(N-2)(\pi G(\tau) + (1-\pi))^{N-3}(1-G(\tau))g(\tau) \\
&= g(\tau)(N-1)(\pi G(\tau) + (1-\pi))^{N-3}\left[-(\pi G(\tau) + (1-\pi)) + (N-2)(1-G(\tau))\pi\right]
\end{aligned}
$$

. Here we at last see the two competing forces that govern the response of prices to increasing search. On the one hand, increasing search marginally increases the incentive to raise prices because a firm worries less about those consumers who are likely to have found a better price, and so doesn't have to include those consumers in its pricing plan. This effect is stronger for those consumers who have a low valuation of the firm's product, i.e. $1 - G(\tau)$ is high.

On the other hand, increasing search incentivizes firms to lower prices because they are now being considered by more consumers who feel they are in close contention with another product. This means the effects of a change in price are larger because there's a marginal population of near-indifferent consumers. This effect is stronger for those consumers who have a high valuation of the product.

The strength of these two forces over the support of possible valuations, weighted by the density of valuations, determines the effect of increased search on prices. The weighting over the types of consumers and therefore the dominant price-search effect is determined by the shape of the taste distribution. A distribution with a fat right tail sees a larger positive effect because they are able to quickly adapt their pricing strategy to focus more on their higher valuation consumers who are less likely to be in contention. Said another way, they quickly have to worry less about the unenthused consumers for whom they are contending because these consumers quickly find a better alternative: the amount of consumers in contention decreases and they can increase prices.

In the obverse, when the distribution has a thinner right tail, the flight of unenthused consumers is less numerous, and instead the dominant effect is an increase in the amount of consumers who view one's product as in contention with another–increasing effective competition. Consumers added this way are price sensitive, and the effect is to decrease the incentive to lower prices.

Take the two examples of used books and toothpaste. In the case of books, when search is low, producers are worried mostly about selling to the large population of consumers who don't value the product highly at all, because this is a large portion of the population and they rarely connect with the small measure that would value them highly. However, as search increases, they are able to focus less on this population of ambivalent consumers and instead focus on the price-insensitive, high-value consumers who really want the book they're selling. For toothpaste, valuations are very concentrated, meaning that increased search serves to greatly increase the number of consumers that are in contention for the firm, and the firm will see strong pressure to decrease prices and capture a larger market share.

Proposition 3.2.2 is an interesting result that demonstrates the two key forces at play and their relative strength. However, the specific environment of unit consumption and linear utility relies on an assumption of an inflexible total amount demanded by the final consumer and therefore understates any price effects of consumption volume. For example, a consumer might decide to purchase more of a product at certain prices, given it is already choosing its favorite from the basket. The next section will detail the case where utility is multiplicative in taste and quantity and such a volume element will be at play.

3.2.5 The Log Utility Case

Suppose for this section that a consumer has log utility over consumption of the good in question. That is, they have preferences represented by the utility function

$$u(x, y) = \ln(\tau x) + y$$

which leads to the value function

$$V(\tau, p) = \ln(\tau x) + (M - px)$$

After some algebra, the symmetric pricing rule will reduce to

$$p = c\left[1 + \frac{\int_{supp(G)} s(\tau)dG(\tau)}{\int_{supp(G)} \tau s'(\tau)dG(\tau)}\right]$$

. Here the firm exhibits multiplicative markups, and the markup is determined by the sensitivity of a firm's probability of sale to deviations in the price as seen in the denominator. Parameterizing the taste distribution function delivers the following result about the reaction of prices to search in this environment.

Proposition 3.2.4. *If G is $\Gamma(m, r)$ and utility is log in taste and quantity $u = \ln(\tau x)$, then prices can only decrease with search*

Proof. Using integration by parts and the properties of the gamma pdf, one can find that

$$\begin{aligned}\int_0^\infty \tau g(\tau)s'(\tau)d\tau &= -\int_0^\infty s(\tau)g(\tau)d\tau - \int_0^\infty \tau s(\tau)g'(\tau)d\tau \\ &= r\int_0^\infty \tau s(\tau)g(\tau)d\tau - m\int_0^\infty s(\tau)g(\tau)d\tau\end{aligned}$$

One can then use this result in the pricing rules to find

$$
\begin{aligned}
p &= c\left[1+\frac{\int_{supp(G)} s(\tau)dG(\tau)}{\int_{supp(G)} \tau s'(\tau)dG(\tau)}\right] \\
&= c\left[1+\frac{1}{r\mathbb{E}_{\hat{G}}[\tau]-m}\right]
\end{aligned}
$$

$\mathbb{E}_{\hat{G}}[\tau]$ is increasing in z due to lemma 3.2.1 $\square$

This result demonstrates that when utility is log, no matter how thick the right tail of the taste distribution, prices can only decrease with search. The key parameter in this environment for determining the effect of search on prices is now the rate parameter r. The higher is r, the more quickly will prices decrease with search.

The reason for this result is that now the competitive effect is augmented by the fact that the price affects the total demanded by an individual who has chosen to buy from a given firm. This means that firms are more reticent to raise prices, and the variety-sampling effect cannot dominate no matter the shape of the distribution.

Note that the lowest that $\mathbb{E}_{\hat{G}}[\tau]$ can be is $\frac{m}{r}$, which corresponds to the monopoly case and delivers the well-known result that there is no profit-maximizing monopolistic price with isoelastic utility.

3.2.6 The Consumer's Problem

Similar to the firm's problem, this section will detail the consumer's problem, working backward in time to develop the relevant conditions. It will show that the distribution that consumers sample from when they search is precisely the pertinent distribution found in the firm's problem: $\tilde{G}$.

Suppose that once a consumer has chosen their desired good, they have a utility function U and budget constraint BC that deliver, through utility maximization, the indirect utility function $V(\tau,p,\mu)$ and demand function $D(\tau,p,\mu)$ over taste, price and shadow value of the numeraire.

Imagine a consumer that has sampled a basket of goods from the N varieties, $\mathcal{B}$ of size k with typical element $(\tau, p) \in \mathcal{B}$. Their problem at this point consists in choosing

$$\max_{b_j \in \mathcal{B}} V(\tau_j, p_j, \mu)$$

Restricting to symmetric price equilibria as in the firm's problem, this problem is isomorphic to choosing the highest taste valuation from amongst the basket. Therefore, the relevant distribution for the shopper is her highest sampled taste value.

Suppose that a consumer samples j of the N unique varieties with their search technology. Their expected utility, ex-ante taste shocks and given the price, is then given by

$$\int_{supp(G)} V(\tau, p, \mu) d\left[G(\tau)^j\right]$$

which is simply the valuation over the distribution of the maximum of j different draws.

Given that each of the N varieties are equally likely, the probability of drawing exactly j unique varieties from a set of N when k total varieties have been drawn is given by

$$\frac{N!}{(N-j)!N^k} S_2(k, j)$$

where $S_2(k,j) = \frac{1}{j!}\sum_{i=0}^{j}(-1)^{j-i}\binom{j}{i})i^k$ is the Stirling number of the second kind which represents the number of ways to partition a set of size k into j non-empty subsets.

Combining all of the above together, the expected utility of a consumer who engages in shopping in an industry is given by

$$\sum_{k=1}^{\infty} \phi(k; z) \sum_{j=1}^{N} \frac{N!}{N^k (N-j)! j!} \sum_{i=0}^{j} (-1)^{j-i} \binom{j}{i} i^k \int_{supp(G)} V(\tau, p, \mu) d\left[G(\tau)^j\right]$$

The following proposition then obtains when prices viewed are Poisson

Proposition 3.2.5. *When the distribution over prices viewed is Poisson with rate z, then* *the expected value of search $S(p,z)$ of a shopper is given by*

$$\begin{aligned} S(p,z) &= \int_{supp(G)} V(\tau,p) d\left[(e^{-z/N} + (1-e^{-z/N})G(\tau))^N \right] \\ &= \int_{supp(G)} V(\tau,p) d\left[\tilde{G}(\tau;z) \right] \end{aligned}$$

where $\tilde{G}$ is the same as given in the firm's section. Given the increasing stochastic dominance property of $\tilde{G}$ from lemma 3.2.1, it is easy to see that *ceteris paribus*, search utility is increasing in number of prices viewed.

Having in hand the value of shopping to the consumer, it is then easy to set up their objective function wherein they choose how much to search:

$$V(p) = \max_z S(p,z) - \xi(z)$$

where ξ is the cost to the consumer of putting in the search effort to generate z. With the sufficient condition that $\xi(z)$ is convex in z, the following first order condition then pins down the search strategy of a consumer in equilibrium which equates the marginal value of search with its marginal cost:

$$S_z(p,z) - \xi'(z) = 0 \tag{Sz}$$

. Given that the symmetric pricing strategy of firms is a function of z and N, we can then use this condition to implicitly define the search rule of consumers for a given number of establishments, $z^*(N)$.

Figure 3.4 displays the search strategy of consumers for given parameterizations and numbers of firms. The downward sloping lines are marginal benefits to search for different parameterizations. The blue dashed lines correspond to a taste distribution with $m = d = 5$ that is, a unimodal thin-tailed distribution; while the solid black lines correspond to a right tailed distribution with $m = d = 0.5$. Two things are evident from the figure. The first is that a right-tailed distribution of tastes implies that the gains to search are higher,

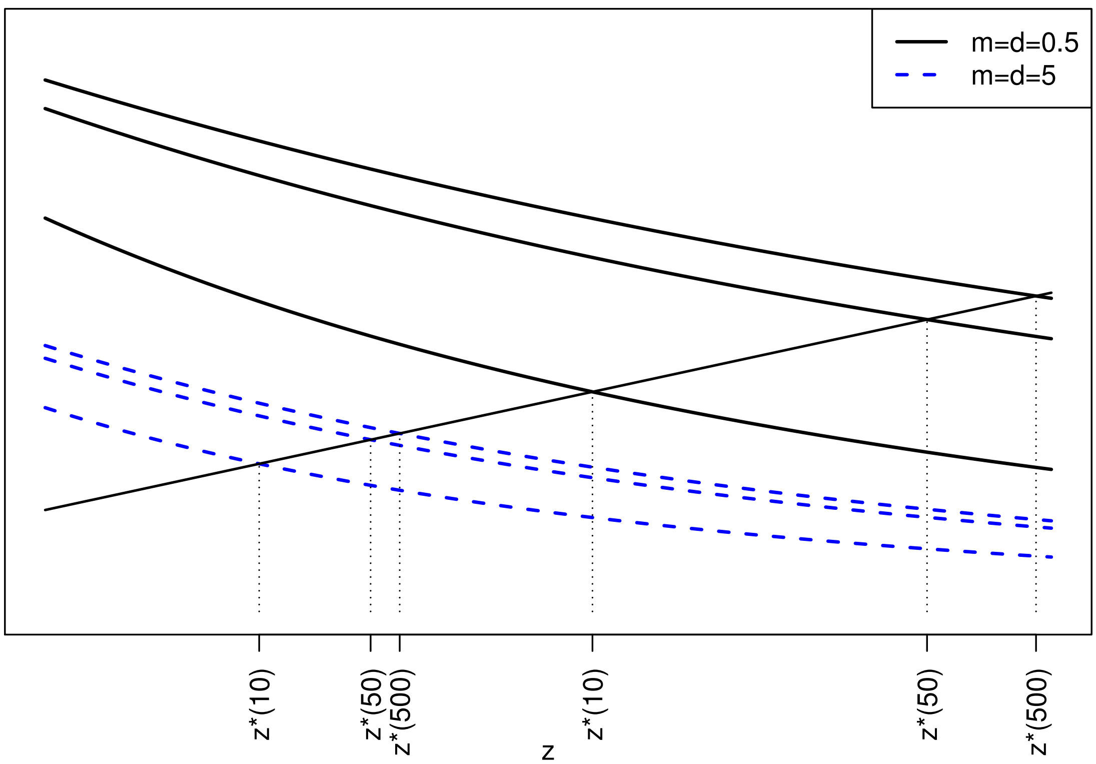


Figure 3.4: Search Strategy

somewhat unsurprisingly. The second, is that the gains to search increase with the number of establishments. This fundamental complementarity between search and number of varieties is also somewhat unsurprising: if there aren't many varieties, the gains to search will be low because the consumer is not effectively sampling as many new products. The size of this complementarity is larger for a right-tailed distribution, and the consumer increases their search by more in response to proliferating varieties in this case.

3.3 General Equilibrium

The strategic behavior of the agents in the previous two sections can be combined with a zero-profit condition for firms to define a general search equilibrium for prices, search behavior and number of establishments. While prices respond by either monotonically increasing or decreasing in response to cheaper search, this will not correspond exactly with the number of establishments in equilibrium. It is possible for the number of establishments to increase as the price decreases due to cheaper search because the impact on surplus of increased demand outweighs that of the lowered price and so firms will enter the market to capture the extra profits.

In order to derive the zero-profit condition, suppose that within a period firms must pay a fixed cost ϕ in order to participate in the market. One can additionally show that the demand for the products of any one firm is given by

$$\frac{1-\tilde{G}(p)}{N}$$

. This can come about through deriving it from the firm's problem or simply noting that firms will divide total demand $1-\tilde{G}(p)$ equally among themselves. The zero-profit condition for firms operating in this market is then given by:

$$(p-c)\frac{1-\tilde{G}(p)}{N}-\phi=0 \tag{ZP}$$

which simply equations the benefits of operating a firm within a period with the costs and closes the model.

Equilibrium in this environment can then be defined as:

Definition 3.3.1. *A symmetric search equilibrium is a triple of prices, search effort and varieties* (p, z, N) *such that:*

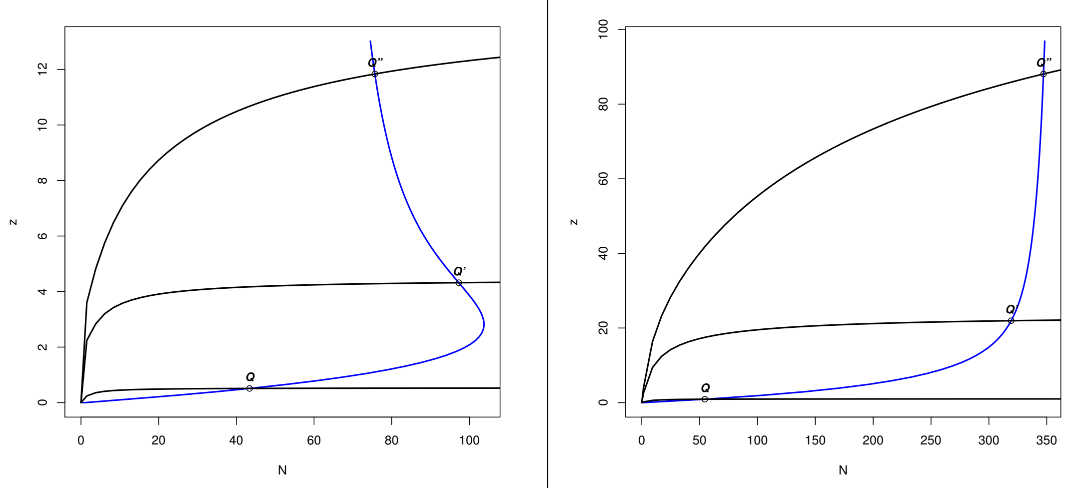
1. *Given the behavior and number of other firms and the search effort of consumers, p solves the firm's pricing rule (FP)*
2. *Given the number of firms and their prices, z solves the consumer's optimal search problem (Sz)*
3. *Firms make zero profits according to (ZP)*

. While the behavior of prices in response to rising search has been documented in the firm section, the response of number of establishments—or the entry and exit of firms—has not. There are two relevant forces present in the zero profit condition that dictate the response of establishments to prices. The first is a demand effect, in that as consumers view more products, they are more likely to find a product that is worth buying and so demand $1-\tilde{G}(p)$ will increase. When $m < 1$ there will be an additional upward pressure on demand as prices fall, increasing the strength of the effect.

The second effect of increased search is the response of prices as seen in the firm section. As search increases, prices will increase for right-tailed distributions ($m < 1$), contributing to per sale profits of firms. In this case, both the demand and price effects move in the same direction and one will see a large increase in varieties from cheaper search. The right panel in Figure 3.5 portrays this case. As search cheapens, prices and demand increase and firms enter the market.

The left panel of Figure 3.5 portray the case when the distribution is not right-tailed. In this environment, the demand and price effects are moving in opposite directions for

Figure 3.5: Equilibria Under Falling and Rising Prices



$m = 5$ $m = 0.5$

firms, and one can see a non-monotonic response of firm numbers to cheapened search as first the demand effect dominates and then the competitive effects of prices take over.

3.4 Conclusion

This paper motivated and explored the impact of falling search costs on both the prices of goods and the entry/exit decisions of firms. It showed that prices can respond to increased consumer search by either rising or falling. The key determinant of the price response lies in the distribution of tastes from which consumers draw: when tastes are right-tailed, search serves to concentrate a firm's buyers among a smaller set of consumers with higher valuations and therefore firms increase prices; while less-dispersed distributions serve to put downward pressure on prices as increased search serves to put a firm into more effective competition with its peers.

The differential responses to prices was shown to feed through into a differential effect on market entry and exit. When consumers sample more products, right-tailed distributions will feature a monotonically increasing number of establishments as variety incentives dominate the market. Alternatively, firm numbers can respond non-monotonically when the distribution is not right-tailed, with demand and price effects working against each other to determine the overall outcome.

There are several intriguing extensions to the paper that are not addressed here. The first is equilibria with different price levels, especially in their outcomes over the business cycle. Kaplan and Menzio (2016) demonstrate the strong feedback effects possible when consumers increase their shopping behavior, and so the possible multiplicity of shopping equilibria with different search costs would be an interesting subject of future study. In addition, the framework described in this paper might shed insight into the cyclicality of markups when firms are heterogeneously efficient and how this is affected by the taste distribution of goods. Lastly, the non-monotonic response of firm numbers to increased search presents an intriguing rationalization for the commonly observed phenomenon of industry shakeouts. Exploring industry shakeouts in a model similar to that in this paper–where consumers gradually become aware of a product–would be an interesting exercise.

The internet is drastically changing the way that consumers shop for their goods. However, as demonstrated in a limited way in this paper, we should not expect the response to be the same in all cases; instead, it will depend on the fundamentals of the market itself how markets react to the cheapness of search.

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